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NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

ROOT PLACEMENT WITH TRANSFER FUNCTION METHODS
(FULL STATE FEEDBACK)

by

Mehmet Ates

December 1988

Thesis Advisor: George J. Thaler

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Root Placement with Transfer Function Methods (Full State Feedback)

by

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1st Lieutenant, Turkish Army
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Submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

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December 1988

ABSTRACT

A design technique for root placement with full state feedback using transfer functions for all-pole plant is presented. A number of examples are presented to demonstrate the procedure with this design technique when all the system states are available to be measured and fedback. The design procedure presented is applicable for linear, time invariant (LTI) systems in the s-domain for continuous time. In order to get the roots at desired locations by transfer function methods very high gains are required. Therefore, the design procedure should place the zeros in offset locations. To obtain some guidelines for offsetting the zeros, the root movement as a function of gain, offset zero locations chosen by matrix methods to put the roots at the desired locations, and arbitrarily chosen offset zero locations are observed on numerous system examples. The obtained guidelines are applied to the all-pole plants.

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- Professor George J. Thaler and other professors who help me widen my knowledge with Patient, and Professional Assistance.
- Turkish, U.S. fellow students , and other International friends for their company.

I. INTRODUCTION

Feedback control is familiar to engineers in all disciplines. In its simplest form, feedback control is nothing more than using the present condition, or "state", of a system to influence its condition in the future.

The advantages of state feedback control can be summarized in four points [Ref. 1]:

1. Assuming that controllability and observability conditions are satisfied, the transient time response of the system can be easily controlled and adjusted.

2. The sensitivity of the system to plant parameter variation is reduced.

3. Rejection of disturbance and noise signals is improved.

4. Steady state errors may be eliminated or reduced.

These benefits are not free. The penalty for using feedback control may include disadvantages such as:

1. System complexity increases because additional sensors may be required to measure the feedback states.

2. Sensors contribute to an increase in cost, size, weight and measurement noise.

3. Closed loop gain is generally lower than open loop gain.

Despite these potential drawbacks, feedback systems are widely used in all engineering fields. In many systems it is desirable to use one or more compensators in feedback rather than in cascade. The decision to use a feedback scheme rather than a cascade compensator is some times a matter of convenience, sometimes a matter of necessity, and for some problems it can be shown that a feedback compensator will do a better job. The purpose of feedback is primarily that of stabilizing the system and providing damping of the system response to the command input, R. Cascade compensation and feedback compensation are entirely equivalent when the system is linear and only basic specifications are considered, that is, they are equivalent in that either can be used and there is no mathematical reason for a preference [Ref. 2].

The basic form for any control system is illustrated in Figure 1 on page 2. It is the responsibility of the design engineer to determine any or all of the items designated in this schematic. In this research, emphasis is placed on the feedback gain "black box" shown in Figure 1 on page 2. The root placement techniques are the methods most commonly used in practice to determine feedback gains.

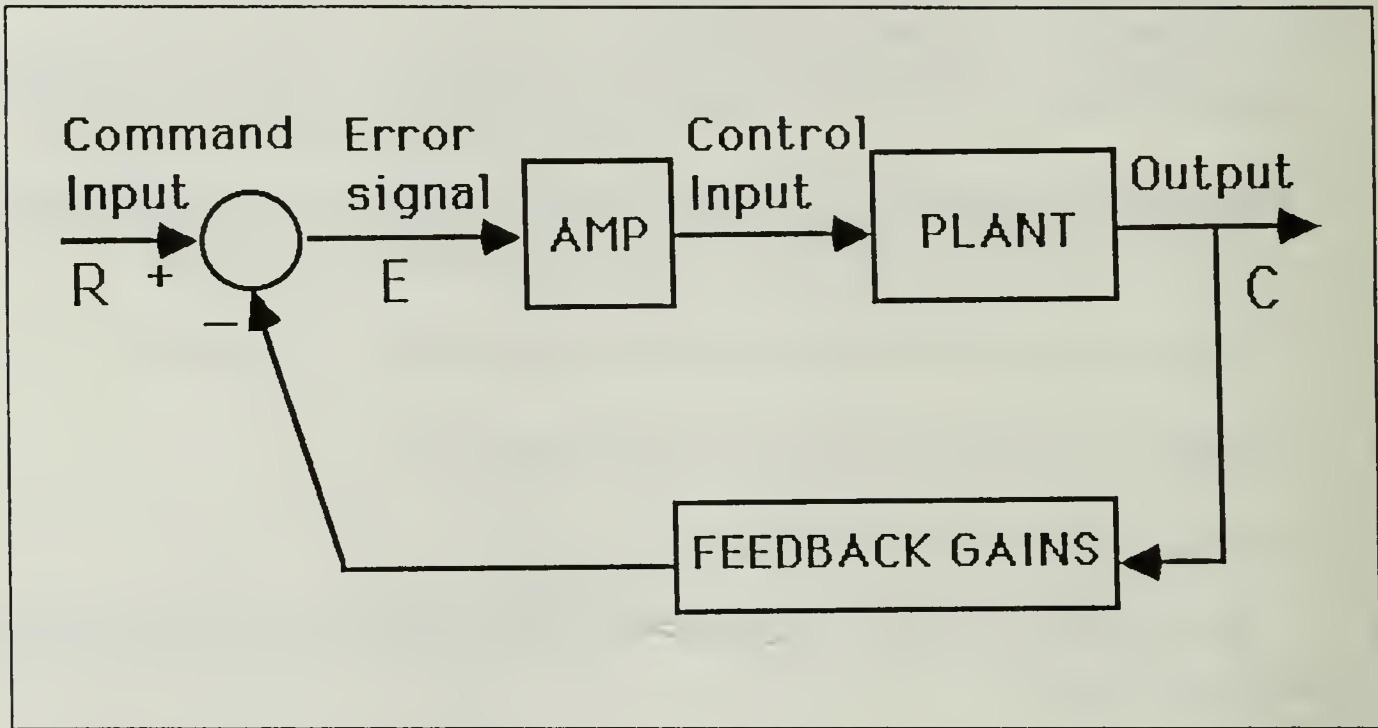

Figure 1. Basic Control System

Root placement with transfer functions is a method of combining state feedback with the Bode method [Ref. 3]. This method uses block diagram manipulations to combine states which are fed back into a feedback transfer function H(s). By adjusting the gain in the forward transfer function, the $|G(s)|$ curve on the Bode plot can be driven high enough above a specified $|1/H(s)|$ curve on the same plot to enable the feedback function to dictate the system response, per the inequalities used in Bode methods (i.e., if $|G(s)H(s)| \gg 1 \Rightarrow |G_{eq}(s)| = |1/H(s)|$). Therefore, theoretically one can specify the characteristic roots for the system using the H(s) transfer function and sufficient forward path gain.

Using root placement with transfer function methods, the system output and its N-1 derivatives are the feedback states. The basic state feedback block diagram is shown in Figure 2 on page 3.

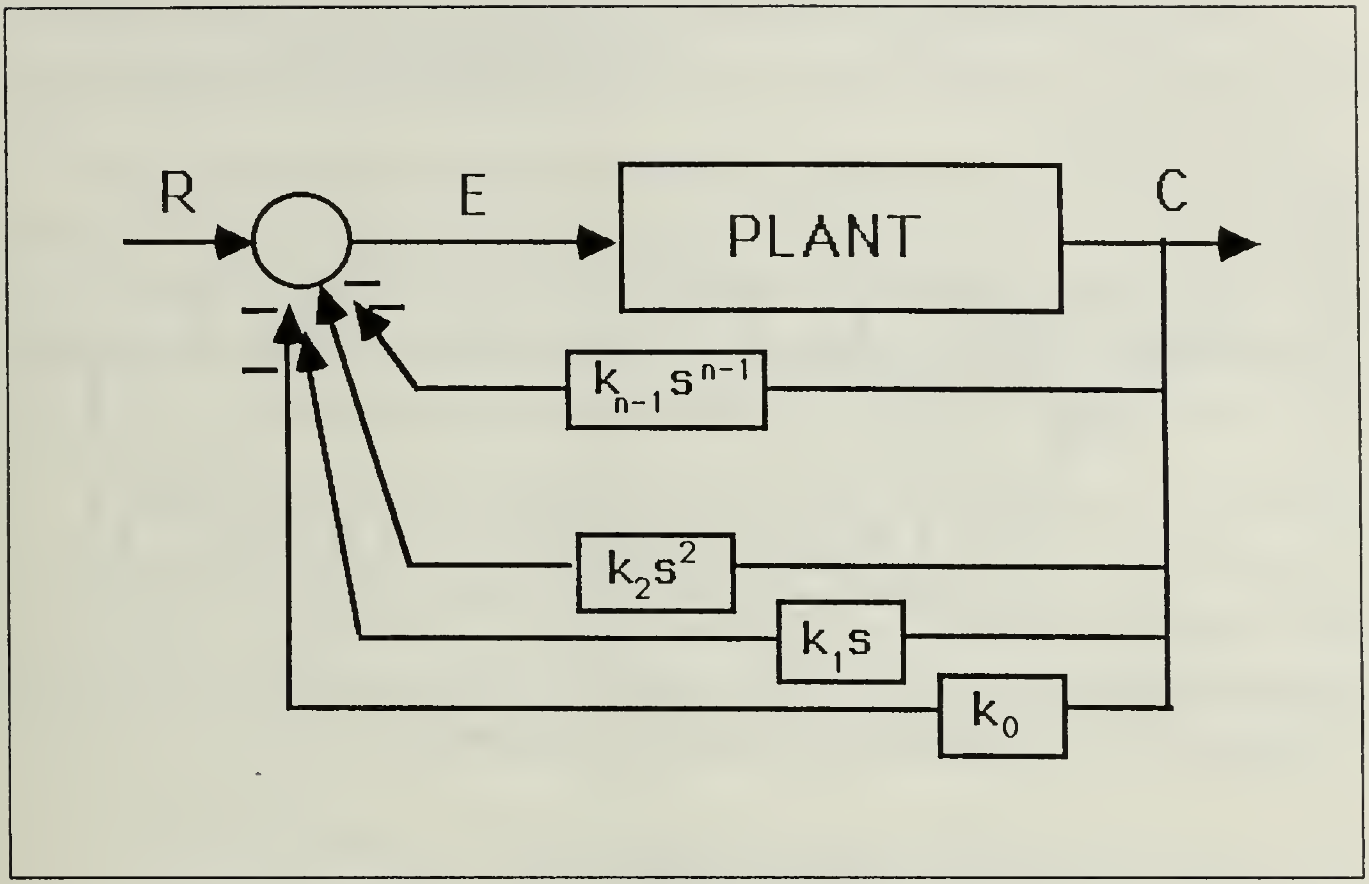

Figure 2. Basic State Feedback Block Diagram

The feedback states can be considered as a feedback polynomial shown in Figure 3.

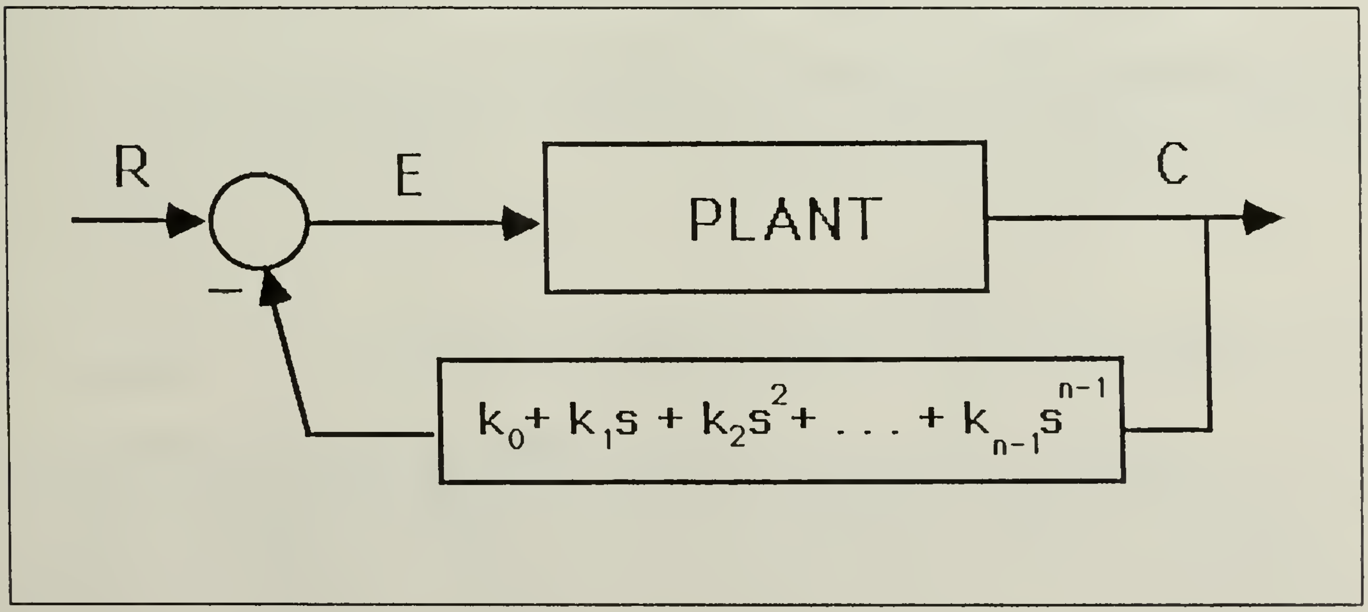

Figure 3. State Feedback Considered As A Transfer Function, H(s).

This polynomial defines the feedback transfer function, H(s). Figure 4 shows that we can preserve the unity feedback by placing the coefficient of the zeroth derivative, k_0 in the forward path.

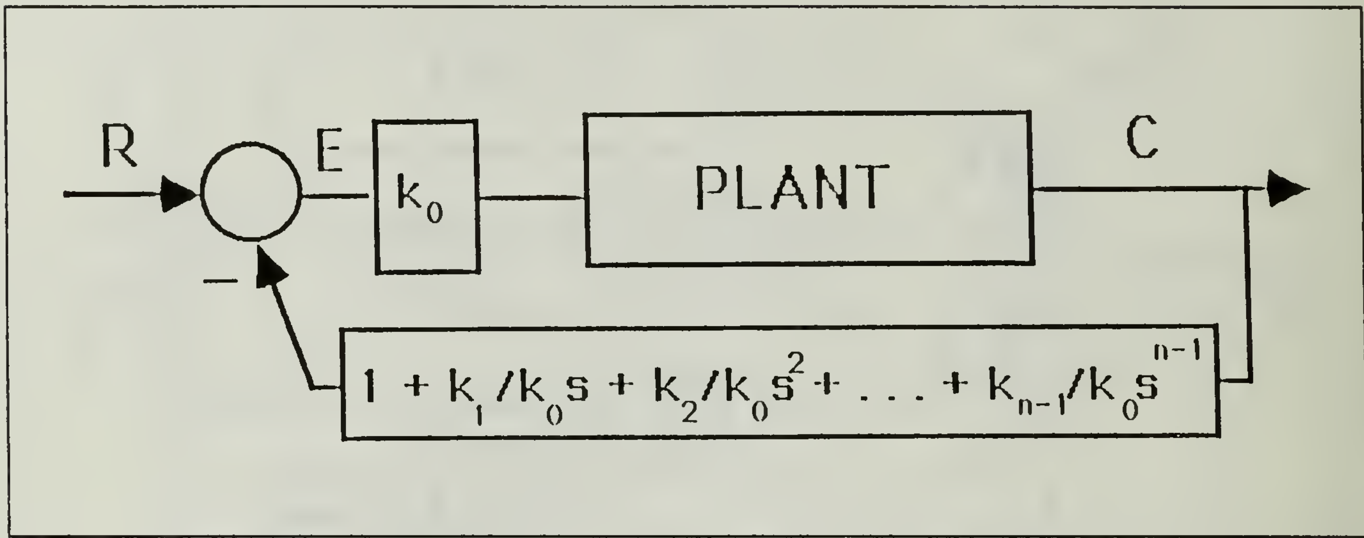

Figure 4. Rearrangement To Preserve Unity Feedback.

Consider that the all-pole plant in general form:

$$G(s) = \frac{K}{s^n + A_{n-1}s^{n-1} + A_{n-2}s^{n-2} + \ldots + A_1 s + A_0} \tag{1.1}$$

and from Figure 2, the feedback transfer function is:

$$H(s) = k_{n-1}s^{n-1} + \ldots + k_1 s + k_0 \tag{1.2}$$

Then the characteristic equation is:

$$1 + G(s)H(s) = 0 \tag{1.3}$$

$$s^n + (A_{n-1} + Kk_{n-1})s^{n-1} + \ldots + (A_1 + Kk_1)s + (A_0 + Kk_0) = 0 \tag{1.4}$$

The roots of any polynomial are functions of the poynomial's coefficients and conversely the polynomial's coeffients are a function of that polynomial's roots. It is obvious that every root of the system may be located by adjustment of the feedback coefficients. Therefore if N-1 roots of the characteristic equation are chosen, the coefficients of this polynomial are the desired feedback gains.

The G(s)H(s) function can be written in factored form such that

$$G(s)H(s) = \frac{K(s+z_1)(s+z_2)(s+z_3)...}{(s+p_1)(s+p_2)(s+p_3)...} \tag{1.5}$$

(i.e., the factors of H(s) became zeros of the loop transfer function G(s)H(s)). When high gain is used, the roots of the characteristic equation are driven to these zeros. Thus, if we choose the zeros of H(s) to be the desired roots of the characteristic equation, then these roots will be achieved if the loop gain is high enough. Since the system has one excess pole, the unspecified root must be real and moves on the negative real axis towards infinity.

In order to get roots at desired locations by using the transfer function method, very high gains are required. This extremely high loop gain is usually not realizable and drastically changes the error coefficient of the system. Therefore the design procedure should place the **zeros** in offset locations, such that the root locus goes through the desired point. Then the adjustable gain need not be so large in order to place the root at the chosen location.

Now, the problem is how do we choose the offset location?

In order to solve this problem, we have to decide:

1. In which direction must the zeros be moved?

2. How far must the zeros be moved?

3. If the root locus does not go through the desired root point exactly, which offset locations are acceptable? In other words, how close to the desired root point must the root locus be located?

4. Do we need to move all zeros?

In order to find the answers to these questions, a number of typical problems will be studied.

Chapter II presents a design technique for root placement with full state feedback using transfer function methods for all pole plant and the root movement as a function of gain. Offset zero locations using matrix methods is researched to develop rules for offsetting zeros on the root locus in Chapter III. Offsetting the zeros is observed and some guidelines found in Chapter IV. Chapter V gives the applications of the guidelines to all pole plants, and results and conclusion in Chapter VI.

II. DESIGN PROCEDURE AND ROOT MOVEMENT AS A FUNCTION OF GAIN

Designing the feedback coefficients and block gain for this system to place roots at desired locations by using the root placement with transfer function method requires that all of the states are available to be measured and fedback as shown in Figure 5. The designer must specifically locate the N-1 roots. The unspecified root will be real and go to infinity as the gain of the system approaches infinity.

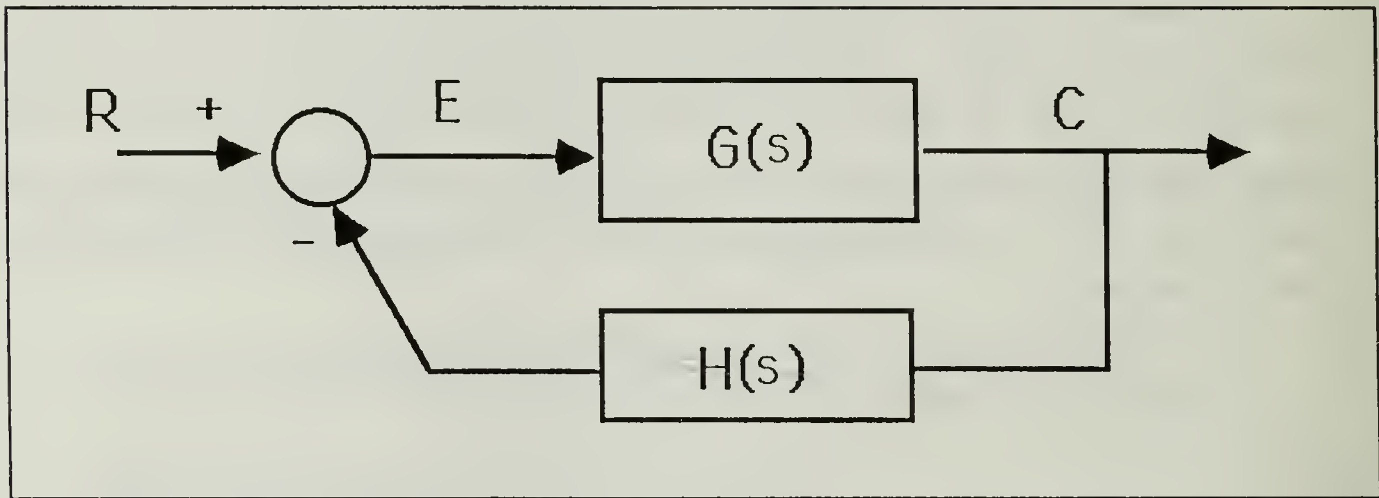

Figure 5. State Feedback Considered As A Function, H(s).

In general, the characteristic equation is:

$$1 + G(s)H(s) = 0 \tag{2.1}$$

where,

$$G(s) = \frac{K}{(s + p_1)(s + p_2)(s + p_3)...} \tag{2.2}$$

$$\begin{aligned} H(s) &= k_0 + k_1 s + k_2 s^2 + ... \\ &= (s + z_1)(s + z_2)(s + z_3)... \end{aligned} \tag{2.3}$$

$$G(s)H(s) = \frac{K(s + z_1)(s + z_2)(s + z_3)...}{(s + p_1)(s + p_2)(s + p_3)...} \tag{2.4}$$

The factors of H(s) became zeros of the loop transfer function G(s)H(s). The roots of the system will move to the desired location when the the gain increases. If we choose the zeros of H(s) to be the desired roots of the characteristic equation, then these roots will be achieved if the loop gain is high enough.

Root movement as a function of gain was studied on a number of the plants. The first step is to completely evaluate the uncompensated system. Then the root movement will be observed when the loop gain increases. It is assumed that the designer knows how to compensate the system such that the compensated system would satisfy the specifications, i.e., stability, accuracy and desired transient response behavior.

A. PLANT 1

The plant transfer function is:

$$G(s) = \frac{K}{s(s+1)(s+2)} \tag{2.5}$$

The order of the plant is three (i.e., N = 3).

The roots have been chosen at -1.000 $\pm$ j2.000 to satistify required system time performance and bandwidth specifications.

So,

$$H(s) = (s + 1 + j2)(s + 1 - j2) = s^2 + 2s + 5 \tag{2.6}$$

The closed loop transfer function is:

$$G_{eq}(s) = \frac{G(s)}{1 + G(s)H(s)} \tag{2.7}$$

where,

$$G(s)H(s) = \frac{K}{s(s+1)(s+2)}\, s^2 + 2s + 5 \tag{2.8}$$

The characteristic equation is:

$$1 + G(s)H(s) = 0 \tag{2.9}$$

$$1 + \frac{K(s^2 + 2s + 5)}{s^3 + 3s^2 + 2s} = 0 \tag{2.10}$$

$$\text{CE: } s^3 + (3 + K)s^2 + (2 + 2K)s + 5K = 0 \tag{2.11}$$

The locations of all roots as a function of block gain, K, are given in Table 1.

Table 1. THE LOCATIONS OF ROOTS AS A FUNCTION OF BLOCK GAIN, K

$s^3 + (3 + K)s^2 + (2 + 2K)s + 5K = 0$			
K	r_1	r_2	r_3
1	-3.32418	-0.37905	±j 1.18264
3	-5.00000	-0.50000	±j 1.65831
5	-6.77326	-0.61337	±j 1.82064
8	-9.55421	-0.72289	±j 1.91417
10	-11.46111	-0.76945	±j 1.94178
30	-31.16502	-0.91748	±j 1.99281
50	-51.09964	-0.95017	±j 1.99738
80	-81.06241	-0.96879	±j 1.99897
100	-101.04995	-0.97502	±j 1.99934
300	-301.01666	-0.99166	±j 1.99992
500	-501.00999	-0.99500	±j 1.99973
800	-801.00644	-0.99687	±j 1.99998
1000	-1001.00499	-0.99750	±j 1.99999
3000	-3001.00164	-0.99916	±j 1.99999
5000	-5001.00111	-0.99955	±j 1.99999
8000	-8001.00000	-0.99968	±j 1.99999
10000	-10001.00000	-0.99975	±j 1.99999
30000	-30001.00000	-0.99990	±j 1.99999
50000	-50001.00000	-0.99995	±j 2.00000
100000	-100001.00000	-0.99997	±j 2.00000

As seen from Table 1, at the beginning the locations of the roots move toward the desired locations rapidly for small increases of the gain. The movement becomes very slow when the roots are close to the desired location. The roots come close to the desired location when the gain is very large. Assuming, we need to place the roots accurately, when the block gain, K, equals 10000, the error of one of the roots is 0.025 %, another is 0.0001 %. For placement of the dominant roots within 1% we need only K = 300.

The system block diagram with state feedback and required block gain, K is given in Figure 6.

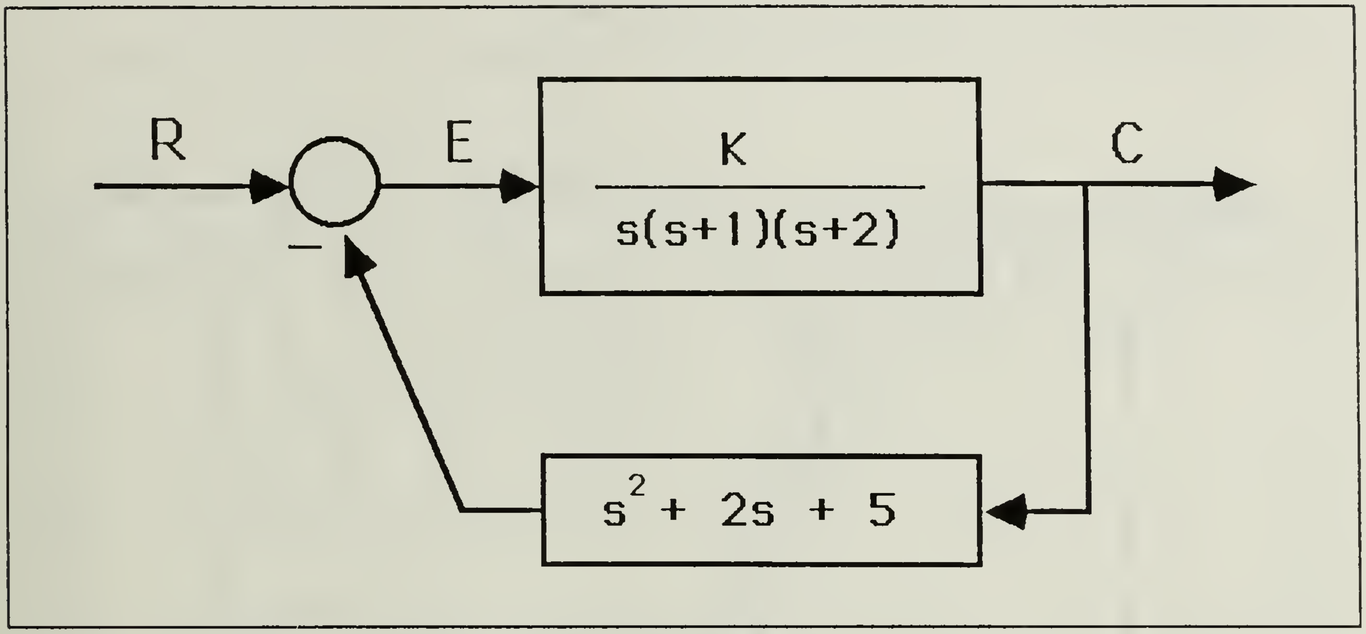

Figure 6. Basic State Feedback Block Diagram

The rearranged block diagram is given in Figure 7.

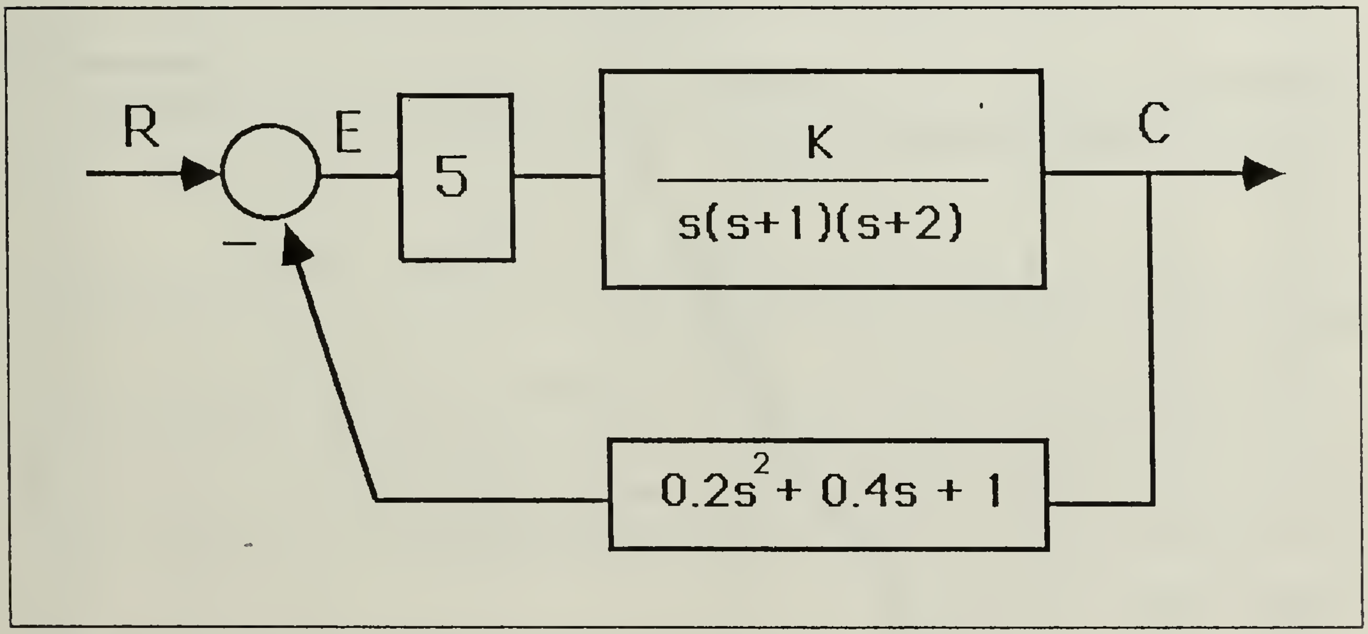

Figure 7. Unity Feedback Preservation

If we rearrange the gains to preserve unity feedback, the feedback plant becomes:

$$H(s) = s^2 + 2s + 5 \tag{2.12}$$

$$H(s) = 5(0.2s^2 + 0.4s + 1) \tag{2.13}$$

$$H(s) = 5(0.2s^2 + 0.4s + 1) \tag{2.13}$$

The rearranged block diagram is given in Figure 8.

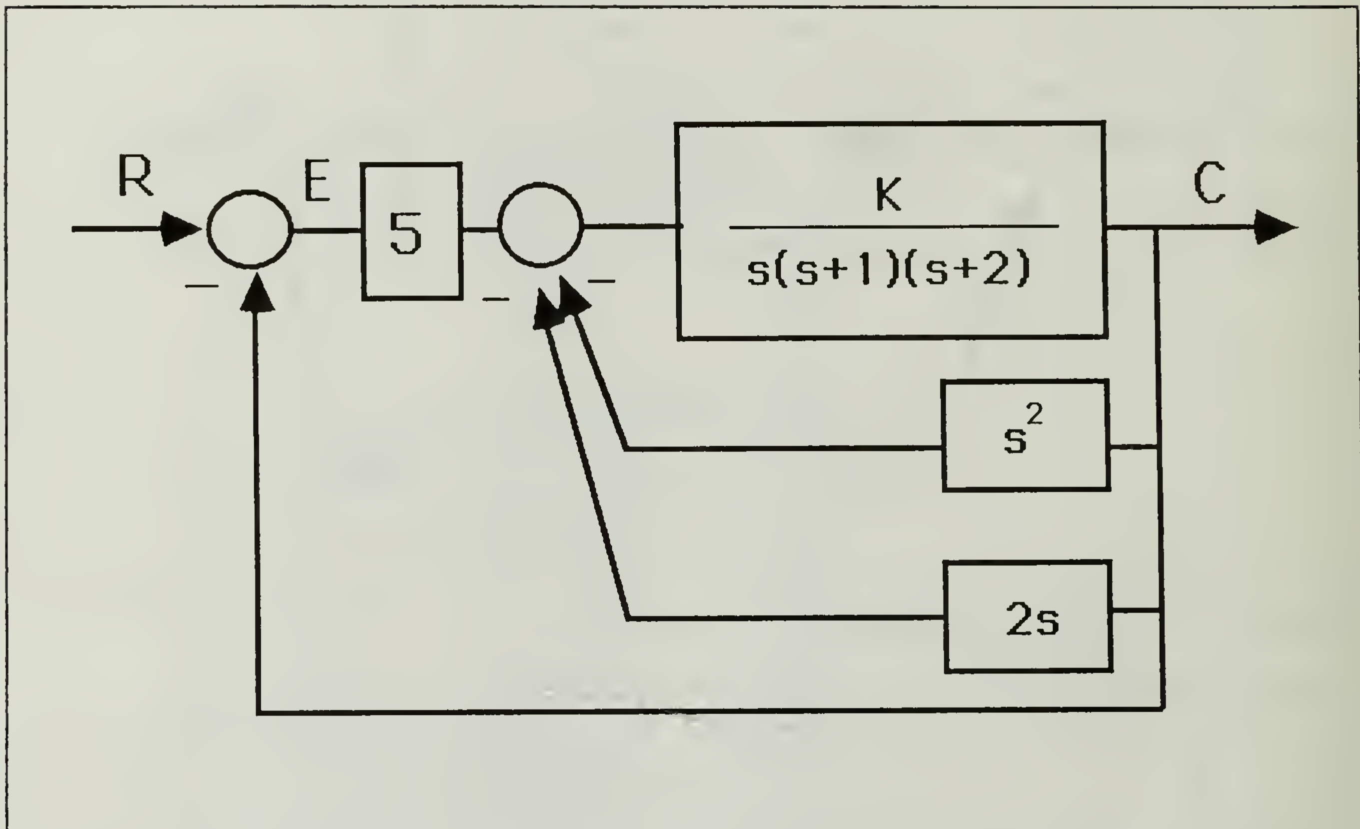

Figure 8. Rearranged Block Diagram

The root locii of the uncompensated and compensated systems are given in Figure 9 on page 11 and Figure 10 on page 12.

The compensated system open loop BODE diagram is given in Figure 11 on page 13. For the compensated system the Gain Crossover Frequency is 1.865 rad/sec, the Phase Margin is 46.64 Degrees, the Phase Crossover Frequency is 24.6 rad sec, and the Gain Margin is 41.75 dB. The closed loop BODE diagram for the compensated system is given in Figure 12 on page 14.

The step response of the compensated system is given in Figure 13 on page 15. The system has a maximum overshoot to a step input at 1.5 sec exceeding the desired steady state value by about 12 % and has about 4 sec settling time.

As seen in figures, the compensated system is stable and the roots are located at the desired points.

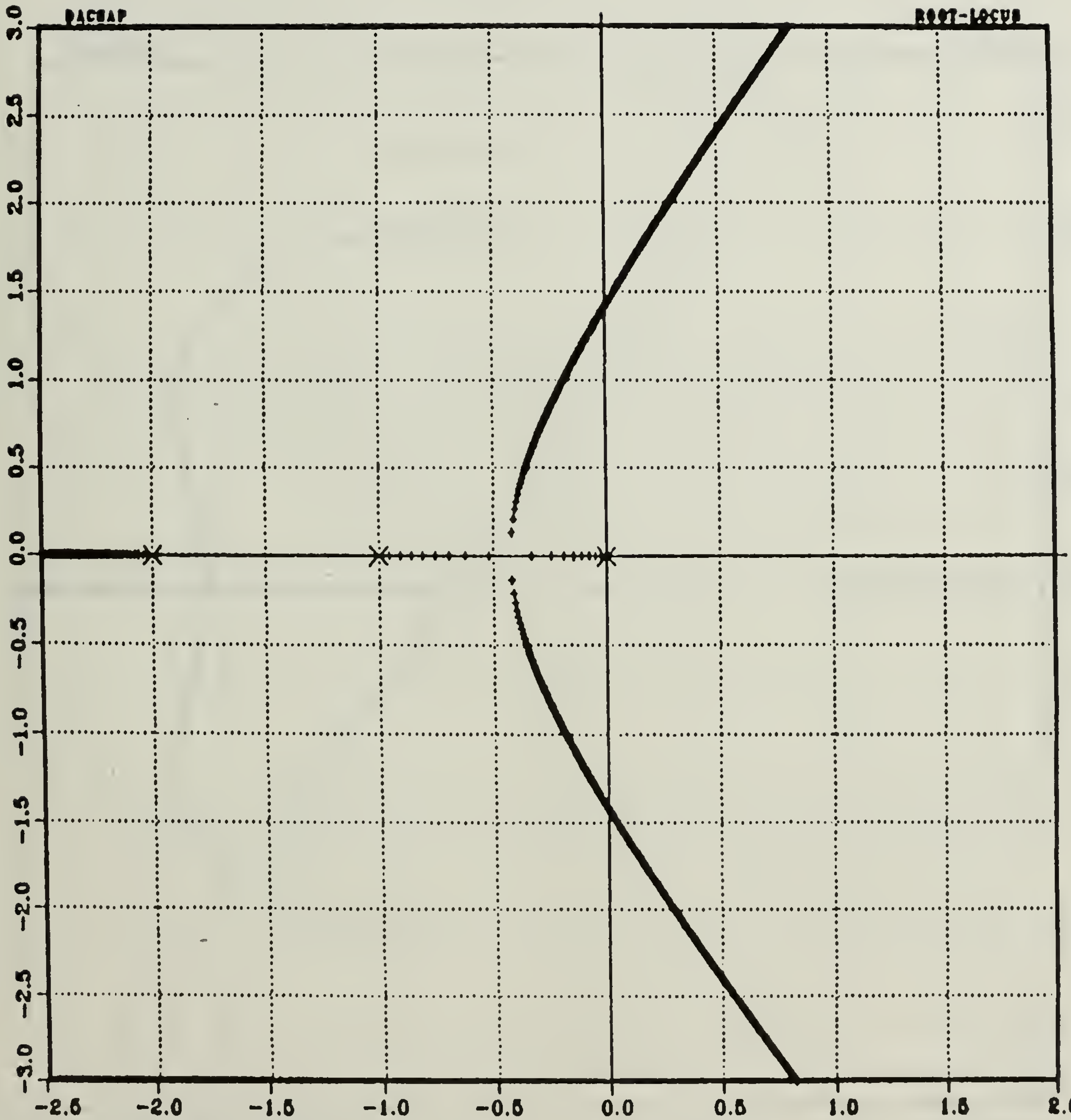

Figure 9. Root Locus of the Uncompensated System

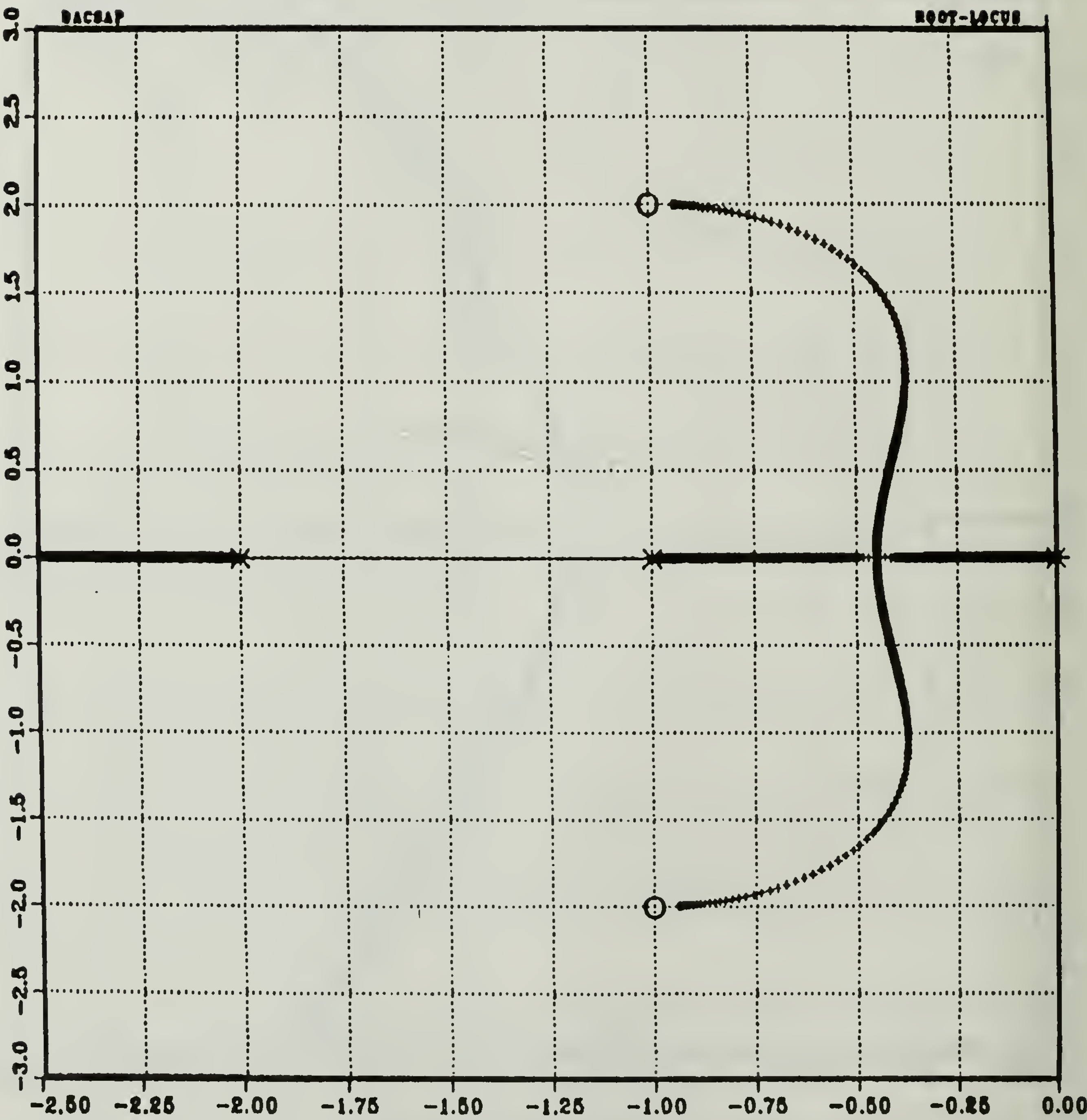

Figure 10. Root Locus of the Compensated System

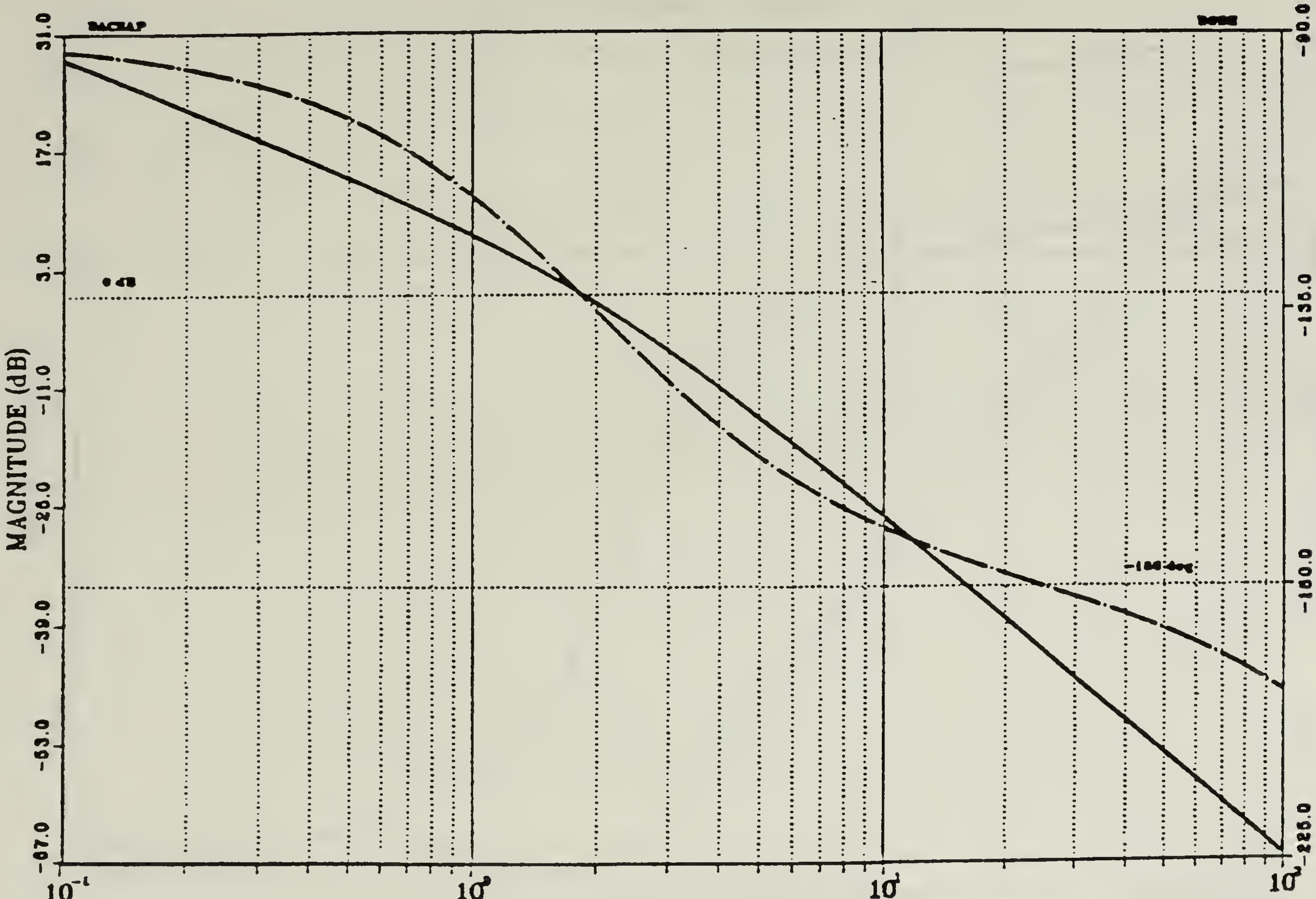

Figure 11. Compensated System Open Loop BODE Diagram

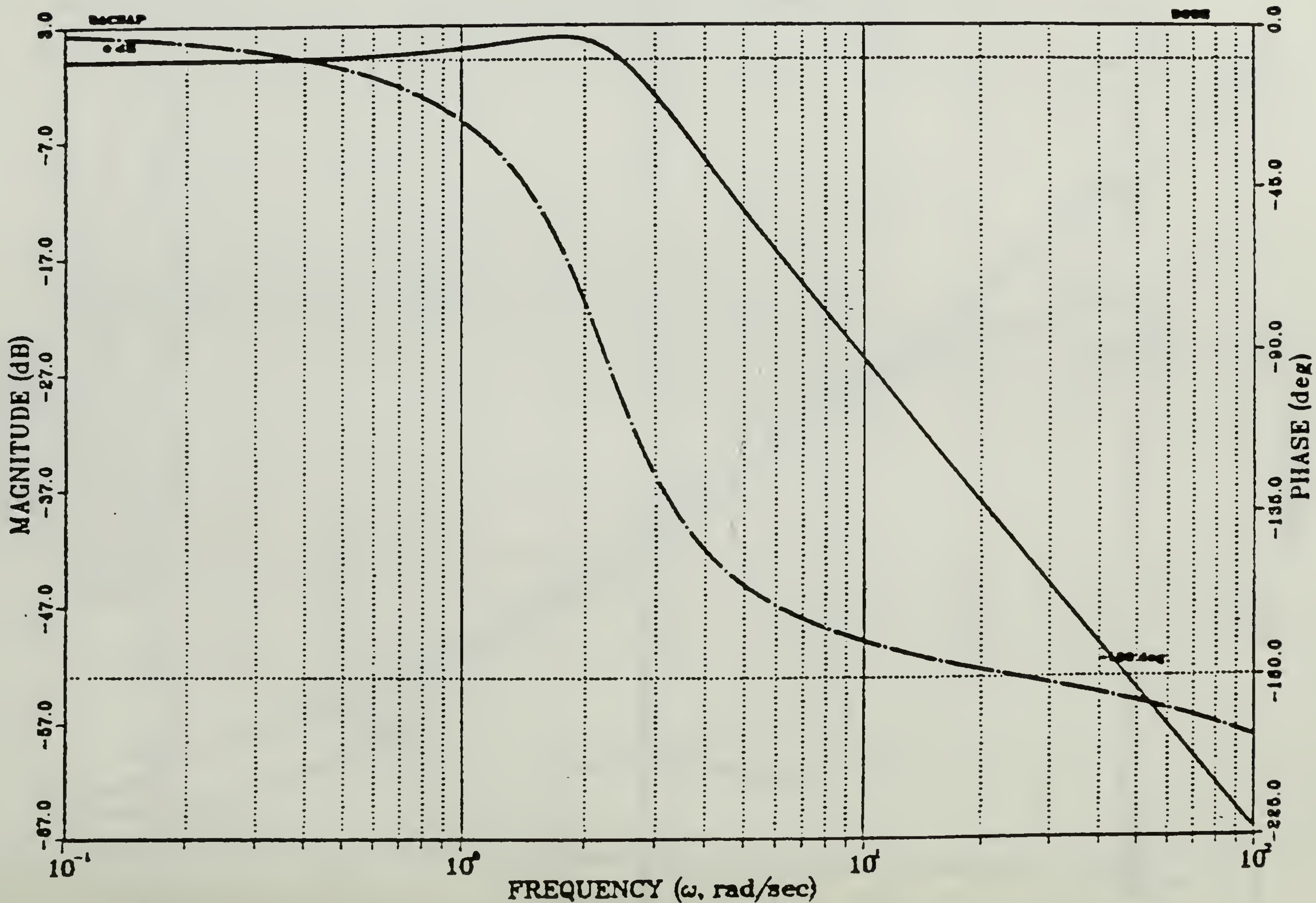

Figure 12. Compensated System Closed Loop BODE Diagram

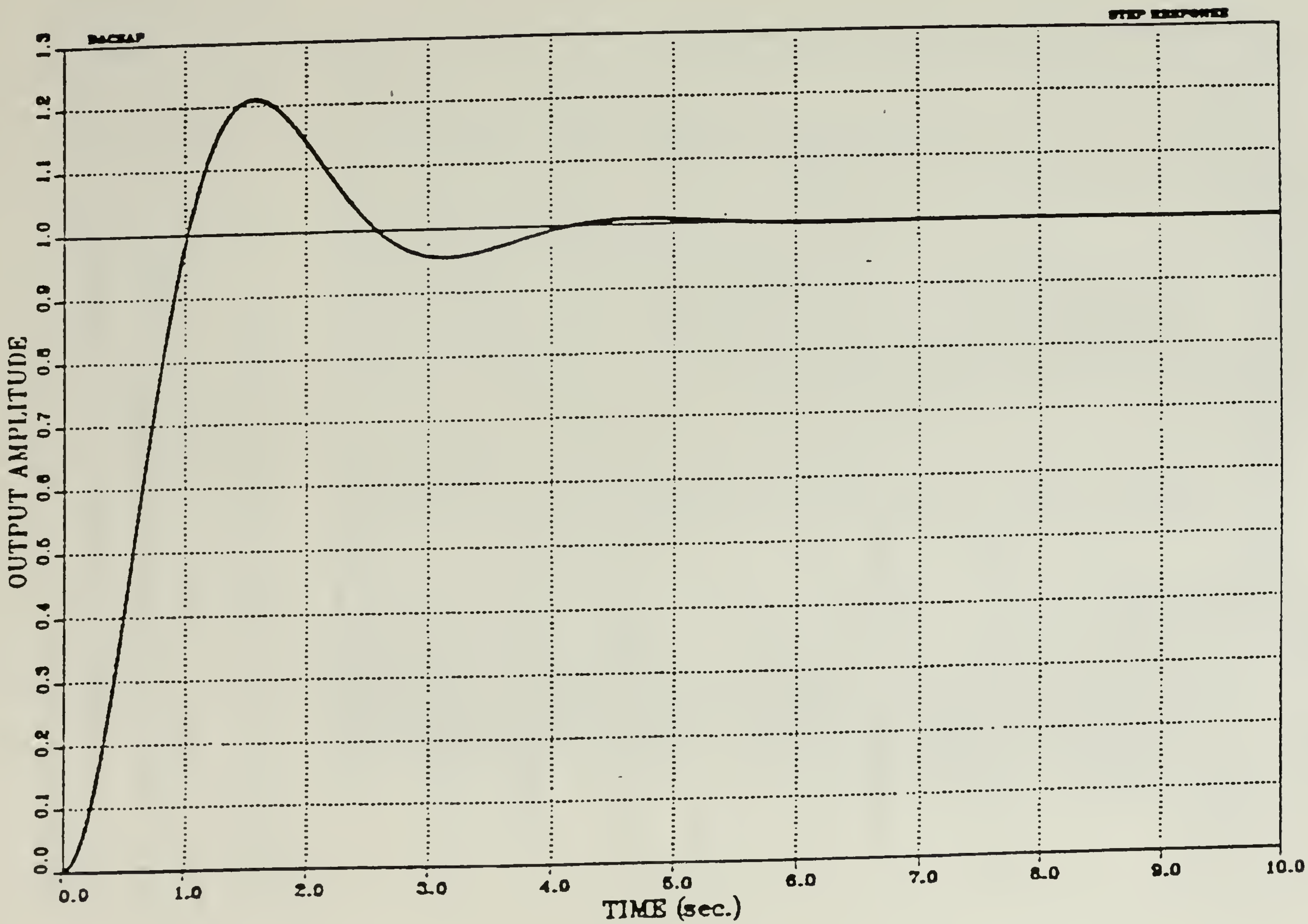

Figure 13. Step Response of the Compensated System

B. PLANT 2

The plant transfer function is:

$$G(s) = \frac{K}{(s+5)(s+3 \mp j5)} \tag{2.14}$$

The order of the plant is three (i.e., N = 3).

Name (N-1) roots:

Desired roots chosen;

-1.000 ± j2.000

So,

$$\begin{aligned} H(s) &= (s+1+j2)(s+1-j2) \\ &= s^2 + 2s + 5 \end{aligned} \tag{2.15}$$

The closed loop transfer function is:

$$G_{eq}(s) = \frac{G(s)}{1 + G(s)H(s)} \tag{2.16}$$

where,

$$G(s)H(s) = \frac{K}{(s+5)(s+3 \mp j5)}\, s^2 + 2s + 5 \tag{2.17}$$

The characteristic equation is:

$$1 + G(s)H(s) = 0 \tag{2.18}$$

$$1 + \frac{K(s^2 + 2s + 5)}{s^3 + 11s^2 + 64s + 170} = 0 \tag{2.19}$$

$$\text{CE: } s^3 + (11 + K)s^2 + (64 + 2K)s + 5K + 170 = 0 \tag{2.20}$$

The locations of all roots as a function of block gain, K, are given in Table 2 on page 17. As the compensated system loop gain is increased, the roots follow the loci. Clearly the unspecified root moves along the negative real axis towards infinity as the loop gain is increased.

Clearly the unspecified root moves along the negative real axis towards infinity as the loop gain is increased.

Table 2. THE LOCATIONS OF ROOTS AS A FUNCTION OF BLOCK GAIN, K

$s^3 + (11 + K)s^2 + (64 + 2K)s + 5K + 170 = 0$			
K	r_1	r_2	r_3
1	-5.82758	-3.08262	$\pm$ j 4.52824
3	-8.22340	-2.88830	$\pm$ j 3.76225
5	-10.82954	-2.58523	$\pm$ j 3.36495
8	-14.47561	-2.26217	$\pm$ j 3.06426
10	-16.77411	-2.21291	$\pm$ j 2.94178
30	-37.95512	-1.52244	$\pm$ j 2.47248
50	-58.31104	-1.34449	$\pm$ j 2.32276
80	-88.54286	-1.22857	$\pm$ j 2.21995
100	-108.62645	-1.18678	$\pm$ j 2.18162
300	-308.86776	-1.06614	$\pm$ j 2.06644
500	-508.91969	-1.04019	$\pm$ j 2.04168
800	-808.94944	-1.02331	$\pm$ j 2.02573
1000	-1008.95949	-1.02029	$\pm$ j 2.02066
3000	-3008.98644	-1.00681	$\pm$ j 2.00696
5000	-5008.99481	-1.00409	$\pm$ j 2.00418
8000	-8008.99480	-1.002567	$\pm$ j 2.00261
10000	-10008.99590	-1.00205	$\pm$ j 2.00209
30000	-30008.99860	-1.00068	$\pm$ j 2.00069
50000	-50008.99920	-1.00040	$\pm$ j 2.00098
100000	-100008.99999	-1.00030	$\pm$ j 2.00020
500000	-500008.99992	-1.00004	$\pm$ j 2.00004
1000000	-1000008.99999	-1.00002	$\pm$ j 2.00021

Table 2 shows the gain required to move the specified roots to their desired locations and the location of the third unspecified root. When the block gain, K, equals 3000, the error is about 1% which is acceptable. The system block diagram with state feedback and required block gain, K, is given in Figure 14 on page 18.

If we rearrange the gains to preserve unity feedback, the feedback plant becomes:

$$H(s) = s^2 + 2s + 5 \tag{2.21}$$

$$H(s) = 5(0.2s^2 + 0.4s + 1) \tag{2.22}$$

In Figure 15, the unity feedback is obtained for the system by adjusting the forward and feedback gains.

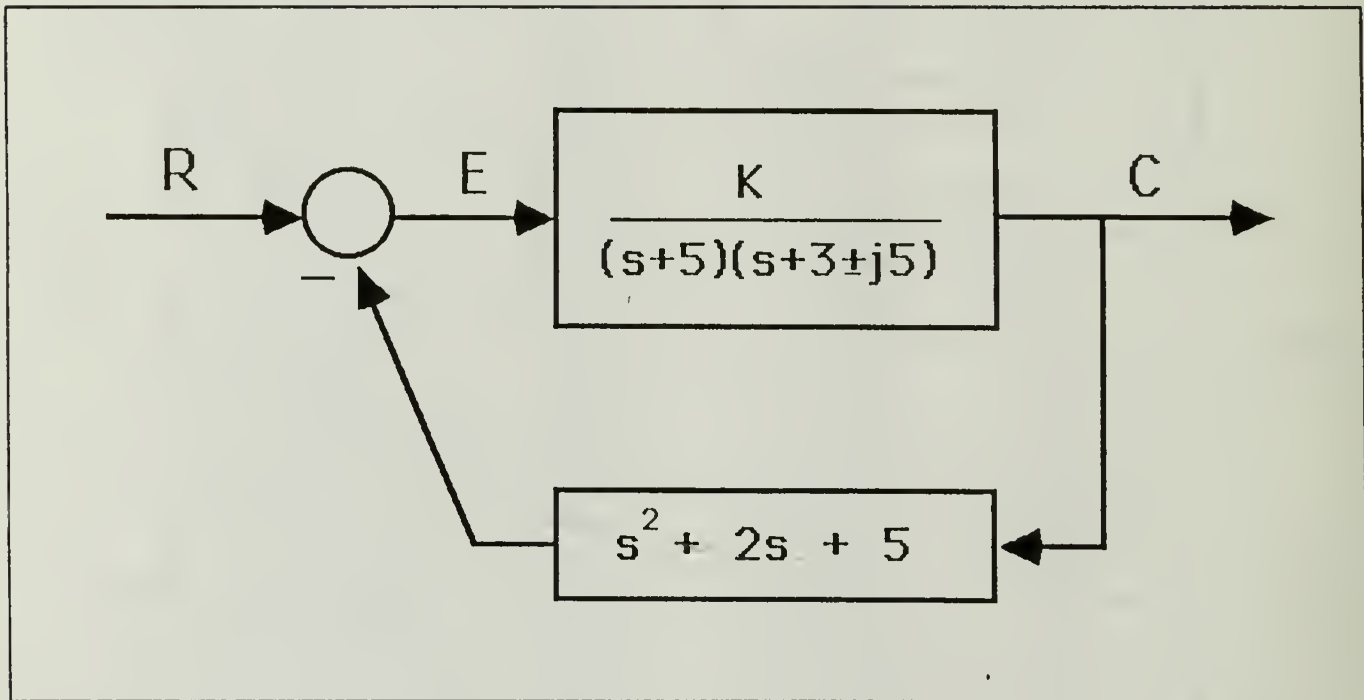

Figure 14. Basic State Feedback Block Diagram

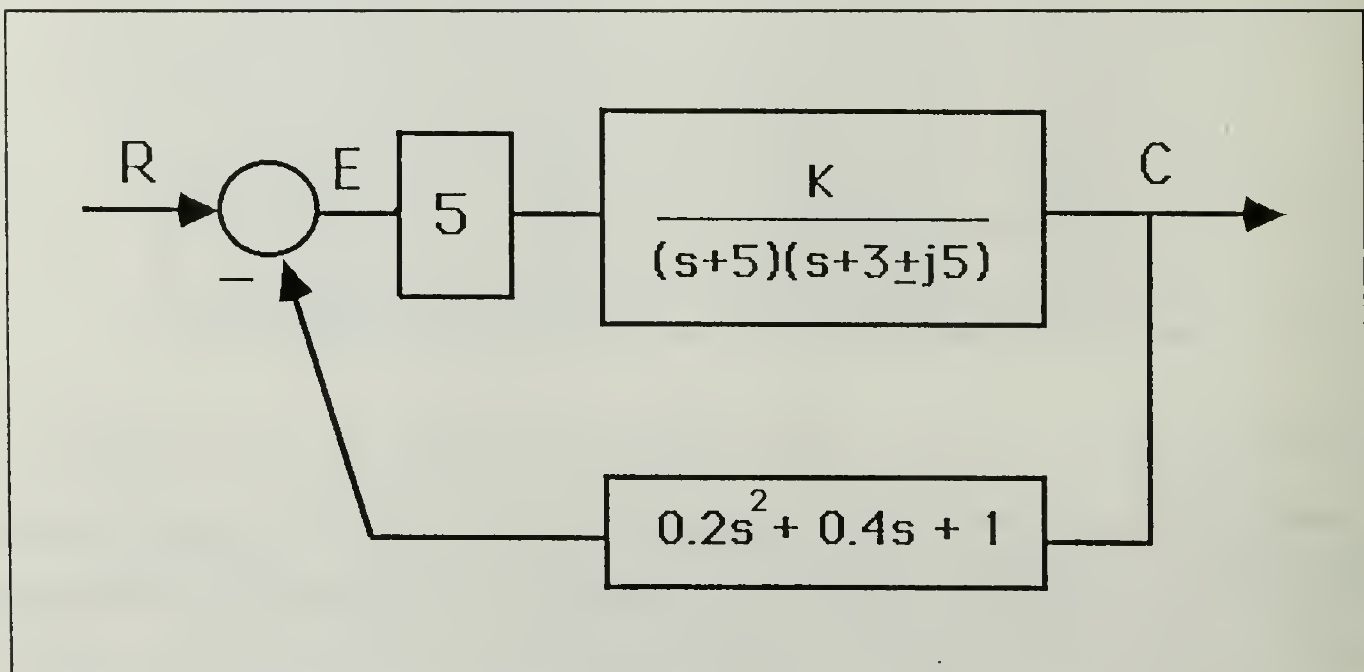

Figure 15. Unity Feedback Preservation

The rearranged block diagram is given in Figure 16.

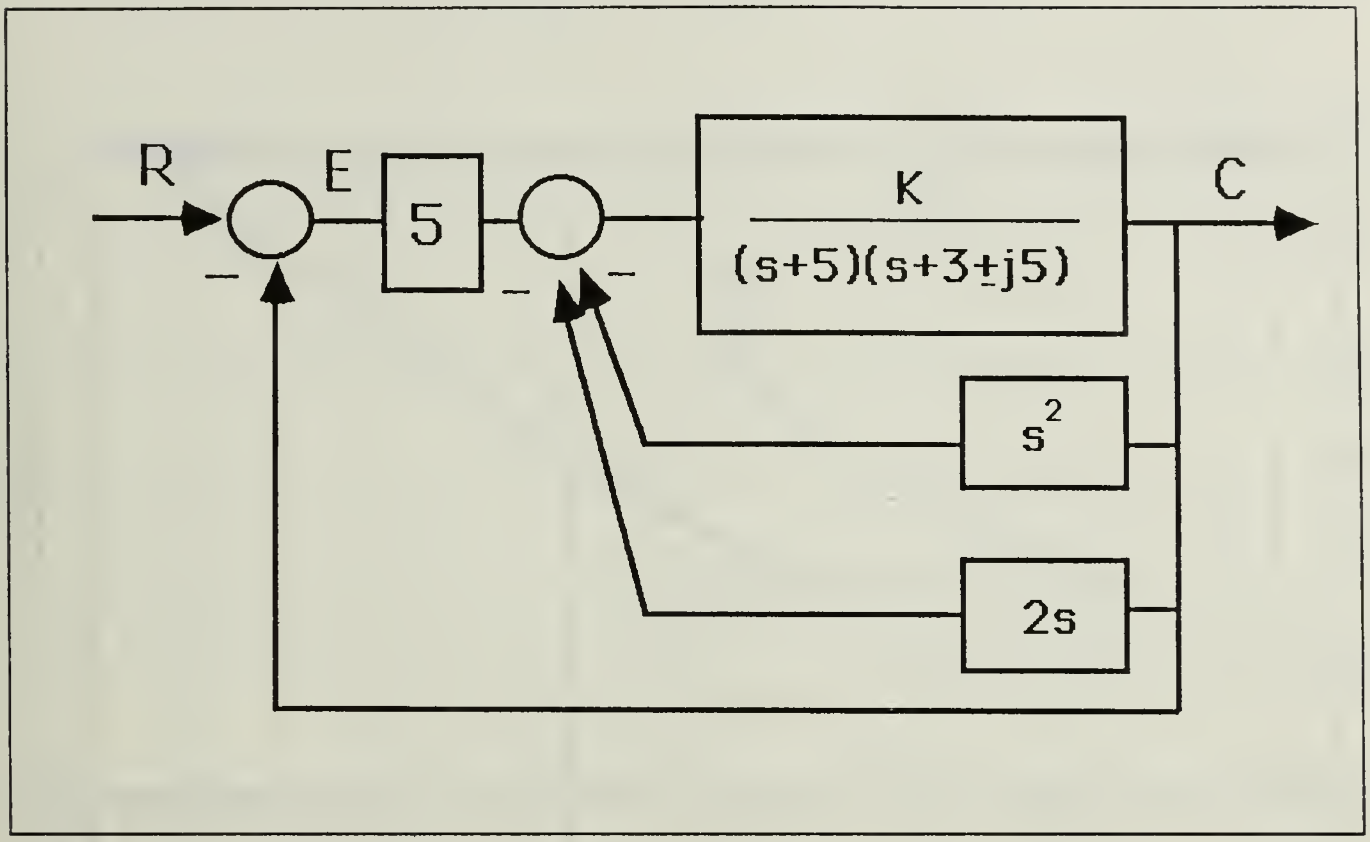

Figure 16. Rearranged Block Diagram

The root locii of the uncompensated and compensated systems are given in Figure 17 on page 20 and Figure 18 on page 21.

The compensated system open loop BODE diagram is given in Figure 19 on page 22. For the compensated system the Gain Crossover Frequency is 1.865 rad/sec, the Phase Margin is 47.64 Degrees, the Phase Crossover Frequency is 78.47 rad/sec, and the Gain Margin is -61.89 dB. The closed loop BODE diagram for the compensated system is given in Figure 20 on page 23.

The step response of the compensated system is given in Figure 21 on page 24. The system has a maximum overshoot to a step input at 1.6 sec exceeding the desired steady state value by about 19 % and has about 4 sec settling time.

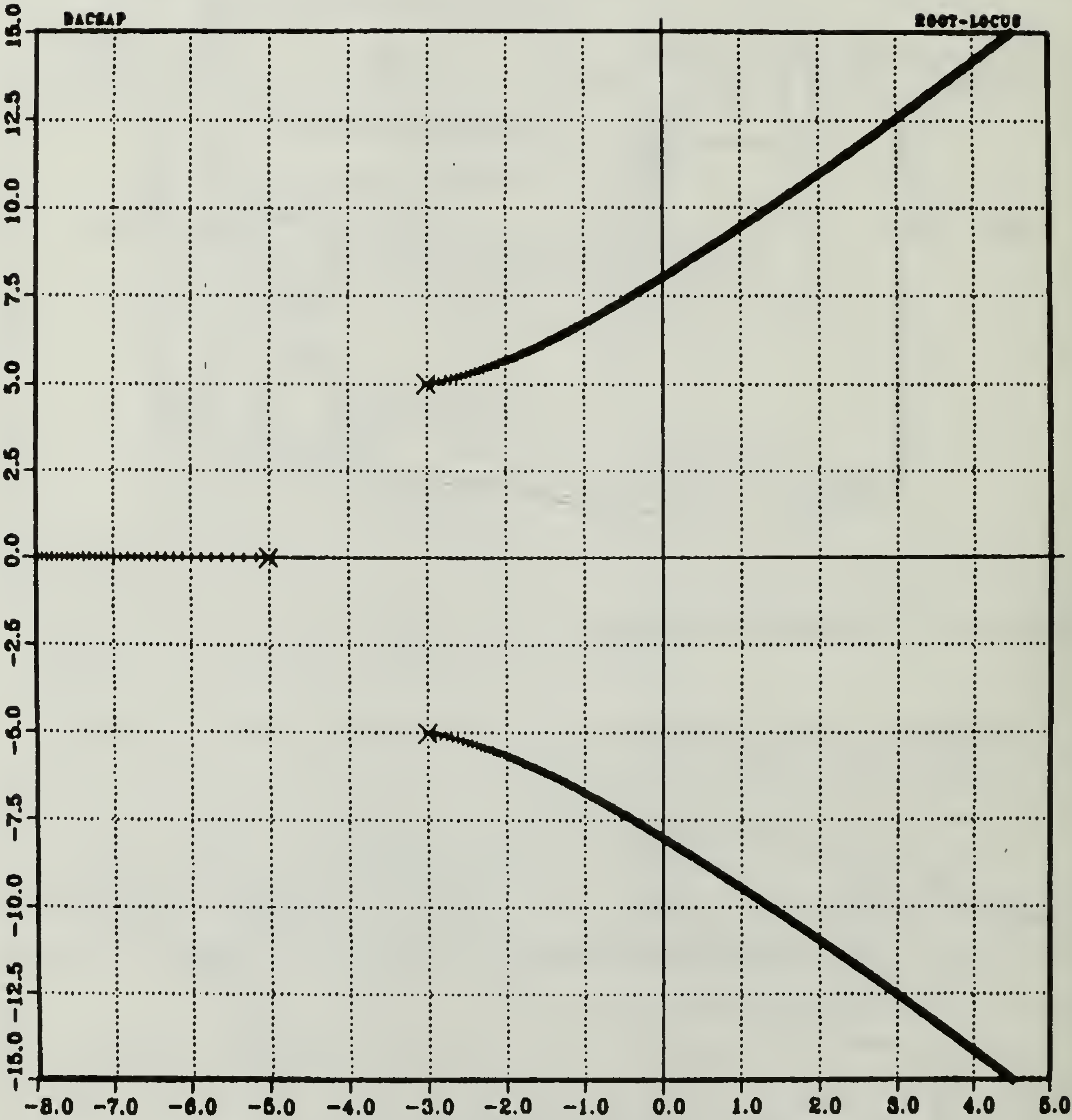

Figure 17. Root Locus of the Uncompensated System

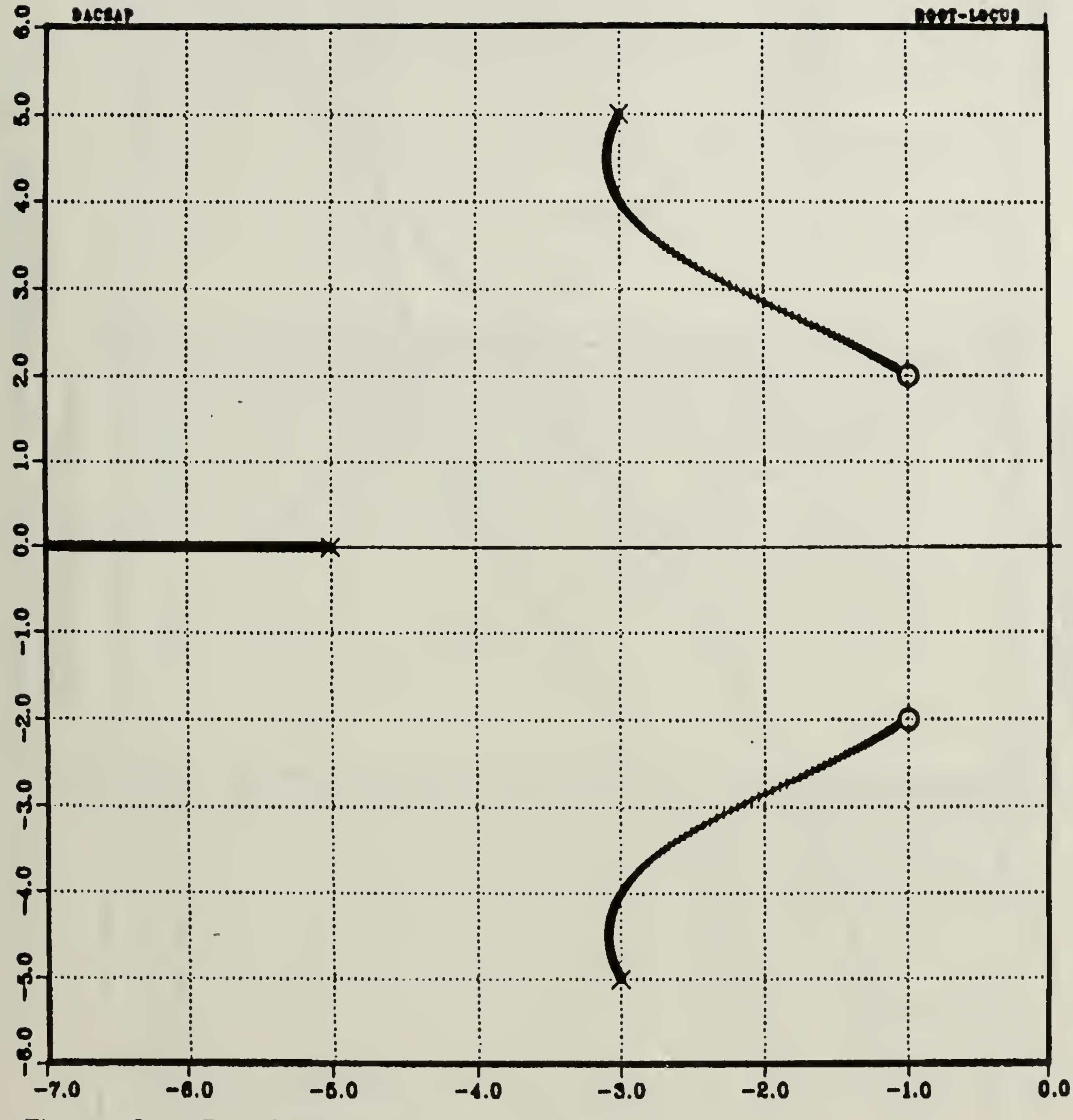

Figure 18. Root Locus of the Compensated System

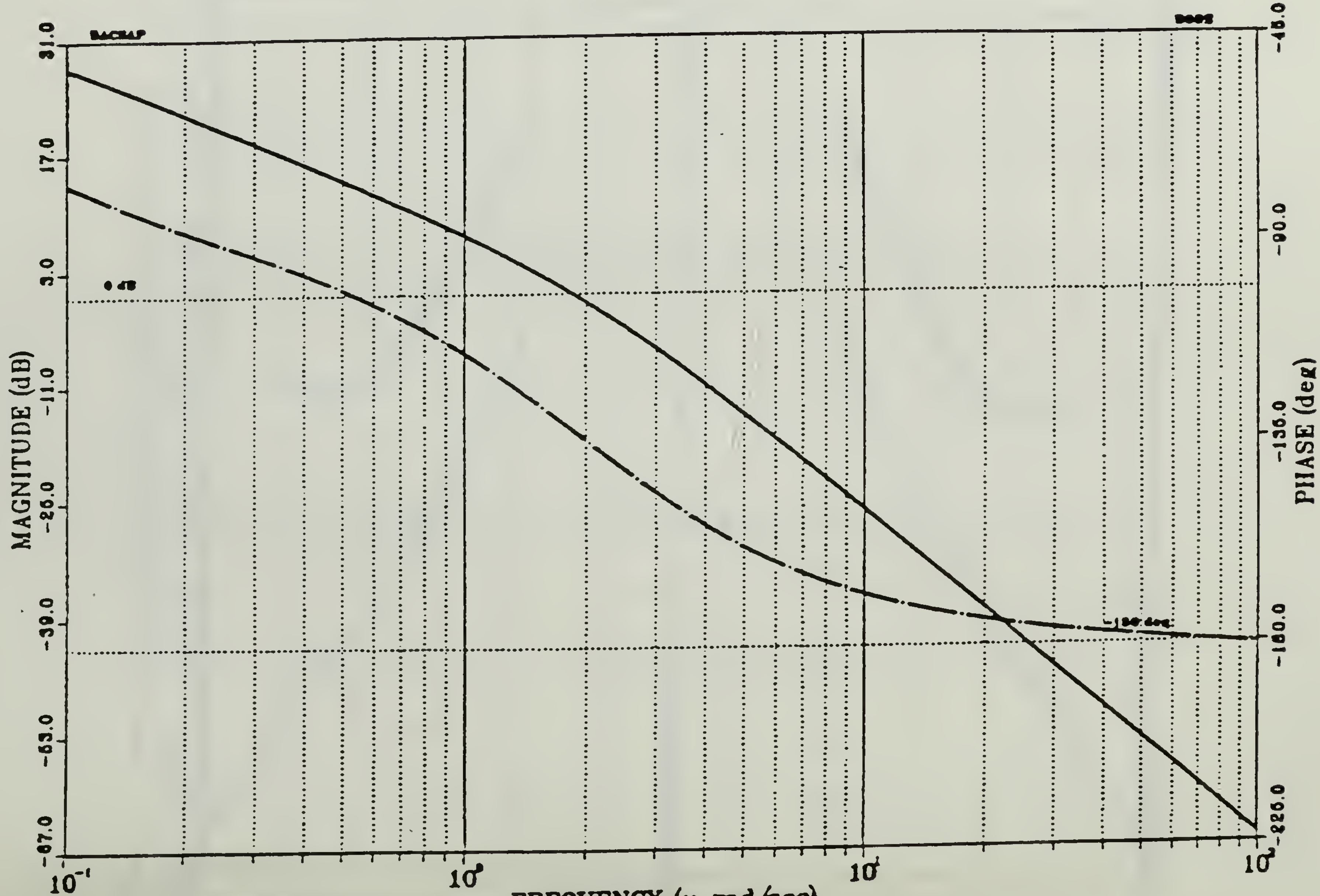

Figure 19. Compensated System Open Loop BODE Diagram

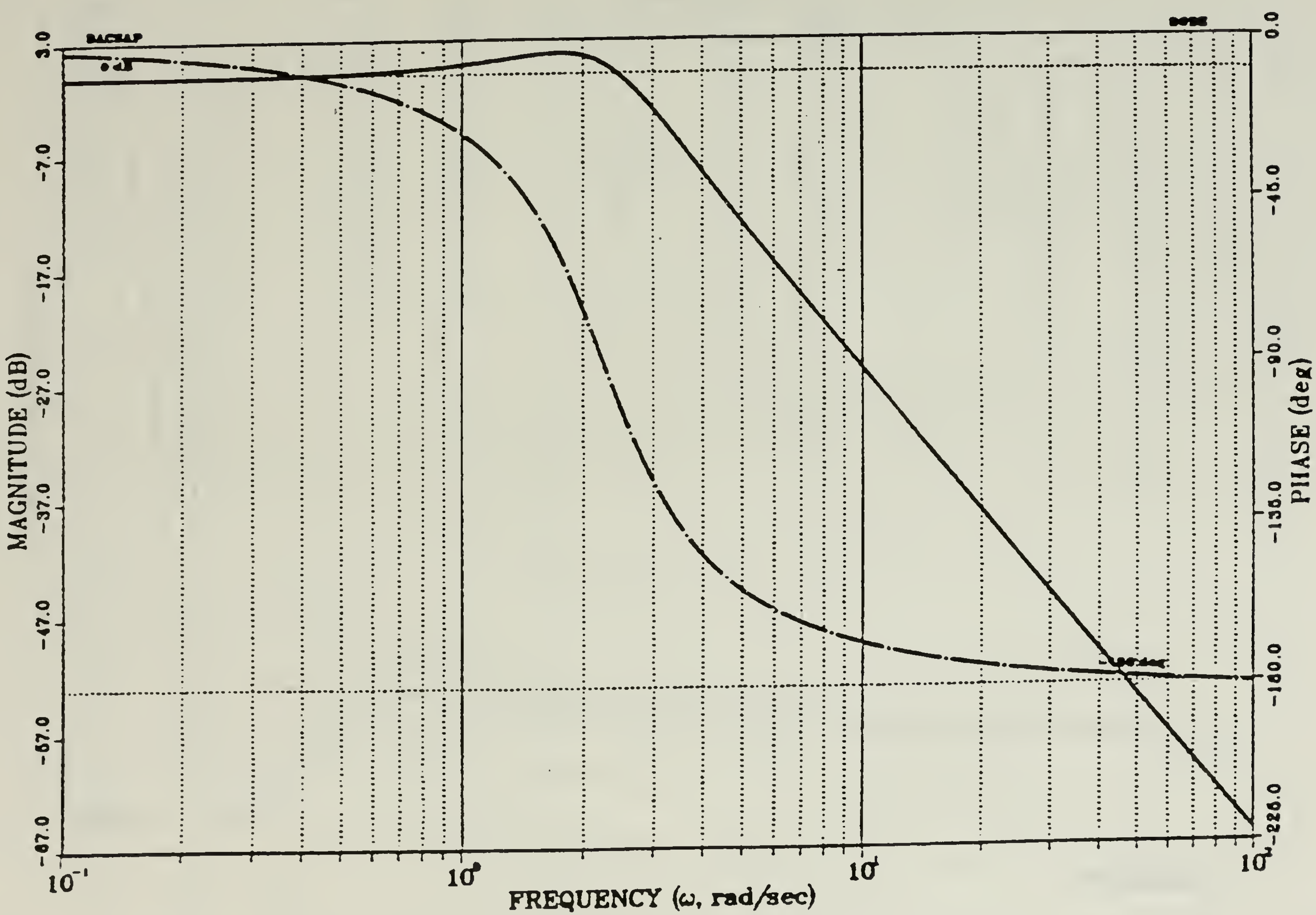

Figure 20. Compensated System Closed Loop BODE Diagram

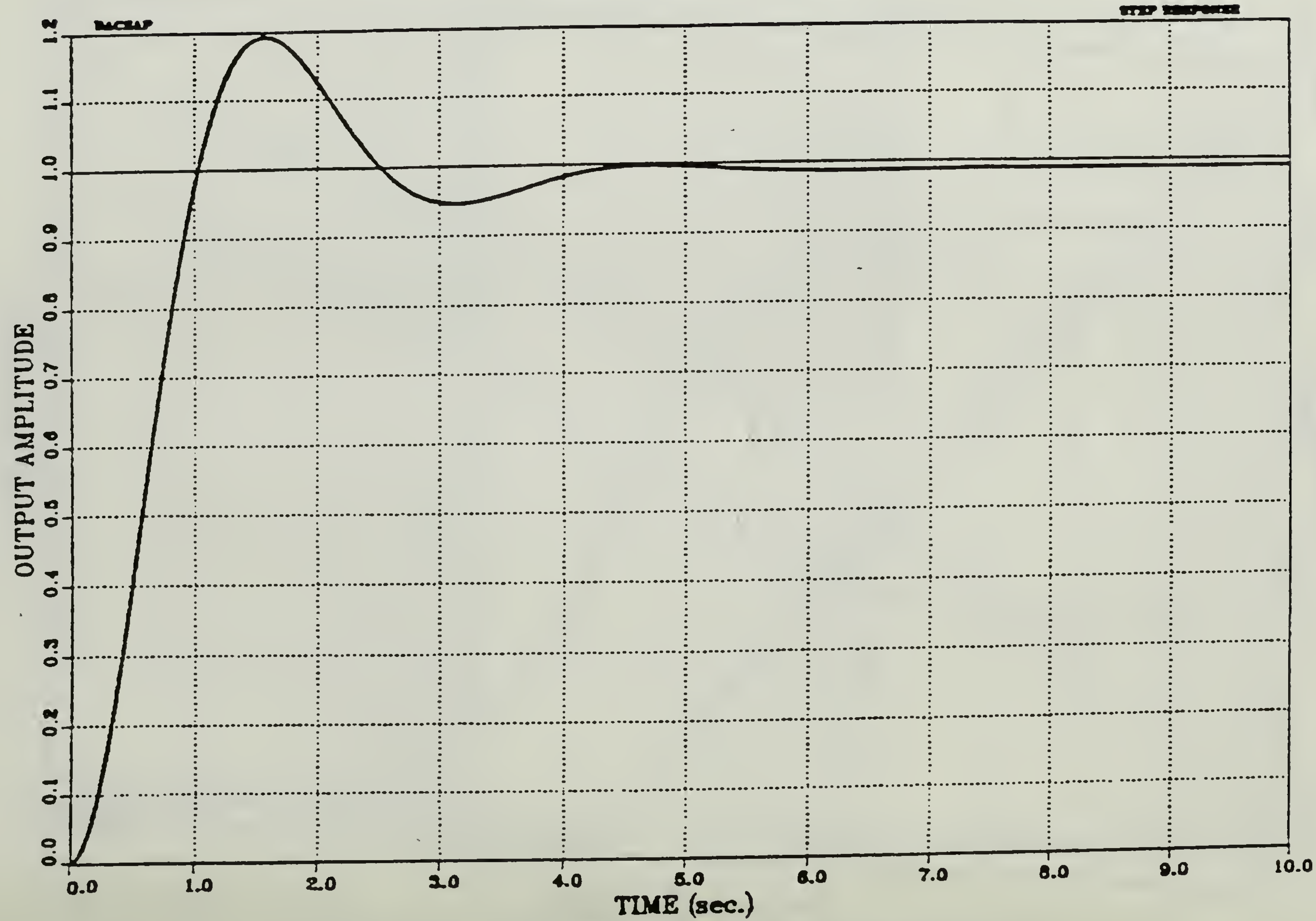

Figure 21. Step Response of the Compensated System

C. PLANT 3

The plant transfer function is:

$$G(s) = \frac{K}{s(s+1)(s+5)(s+10)(s+50)} \tag{2.23}$$

The order of the plant is five (i.e., N = 5).

Name (N-1) roots:

Desired roots chosen;

-3.000 ± j5.000

-15.000 ± j5.000

So,

$$\begin{aligned} H(s) &= (s+3+j5)(s+3-j5)(s+15+j5)(s+15-j5) \\ &= s^4 + 36s^3 + 464s^2 + 2520s + 8500 \end{aligned} \tag{2.24}$$

The closed loop transfer function is:

$$G_{eq}(s) = \frac{G(s)}{1 + G(s)H(s)} \tag{2.25}$$

where,

$$G(s)H(s) = \frac{K}{s(s+1)(s+5)(s+10)(s+50)}\, s^4 + 36s^3 + 464s^2 + 2520s + 8500 \tag{2.26}$$

The characteristic equation is:

$$1 + G(s)H(s) = 0 \tag{2.27}$$

$$1 + \frac{K(s^4 + 36s^3 + 464s^2 + 2520s + 8500)}{s(s+1)(s+5)(s+10)(s+50)} = 0 \tag{2.28}$$

$$s^5 + (66+K)s^4 + (326+36K)s^3 + (605+464K)s^2 + (10500+2520K)s + 8500K = 0 \tag{2.29}$$

The locations of all roots as a function of block gain, K, are given in Table 3 on page 26.

Table 3. THE LOCATIONS OF ROOTS AS A FUNCTION OF BLOCK GAIN, K

$s^5 + (66 + K)s^4 + (326 + 36K)s^3 + (605 + 464K)s^2 + (10500 + 2520K)s + 8500K = 0$					
K	r_1	r_2	r_3	r_4	r_5
1	-61.32	-7.3247	1.1667	±j 5.1300	-0.8634
5	-63.68	-6.8455	0.9388	±j 6.3819	-2.3426
10	-66.77	-5.8024	0.3352	±j 7.3252	-4.0791
50	-96.35	-4.5244	±j 8.0375	-5.2998	±j 4.8740
100	-140.31	-9.2051	±j 7.8755	-3.6376	±j 5.2958
500	-532.39	-13.7075	±j 6.2959	-3.0975	±j 5.0487
1000	-1031.21	-14.3487	±j 5.7321	-3.0475	±j 5.0238
5000	-5030.24	-14.8691	±j 5.1631	-3.0093	±j 5.0047
10000	-10030.12	-14.9345	±j 5.0827	-3.0046	±j 5.0023
50000	-50030.02	-14.9869	±j 5.0167	-3.0009	±j 5.0005
100000	-100030.01	-14.9934	±j 5.0088	-3.0004	±j 5.0002
500000	-500030.00	-14.9991	±j 5.0050	-3.0002	±j 5.0001
1000000	-1000030.00	-14.9999	±j 5.0001	-3.0000	±j 5.0000

Table 3 shows the gain required to move the specified roots to their desired locations and the location of the fifth unspecified root. Again, the movement becomes very slow when the roots are close to the desired location. When the block gain, K, equals 5000, the error is about 1% which is acceptable.

The system block diagram with state feedback and required block gain, K is given in Figure 22 on page 27.

If we arrange the gains to preserve unity feedback, the feedback plant becomes:

$$H(s) = s^4 + 36s^3 + 464s^2 + 2520s + 8500 \tag{2.30}$$

$$H(s) = 8500(0.0012s^4 + 0.00424s^3 + 0.05459s^2 + 0.29647s + 1) \tag{2.31}$$

In Figure 23 on page 27, the unity feedback is obtained for the system by adjusting the forward and feedback gains.

The rearranged block diagram is given in Figure 24 on page 28.

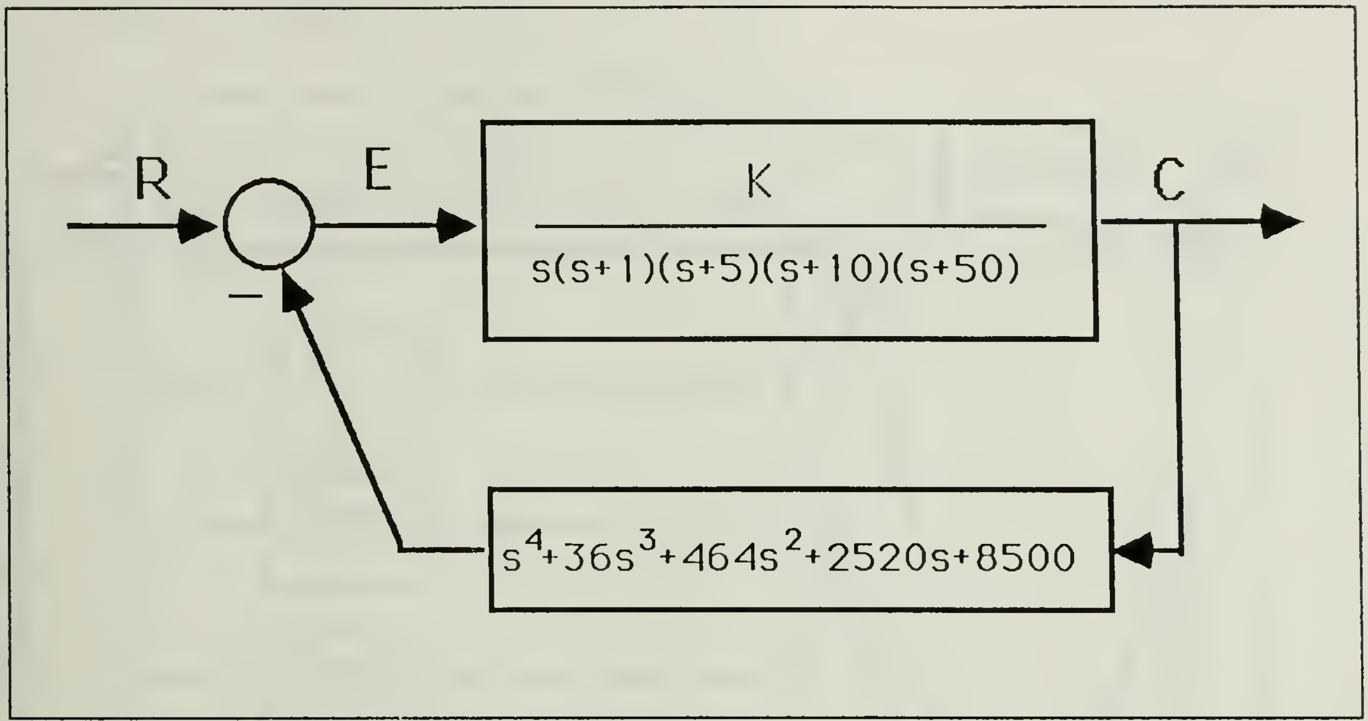

Figure 22. Basic State Feedback Block Diagram

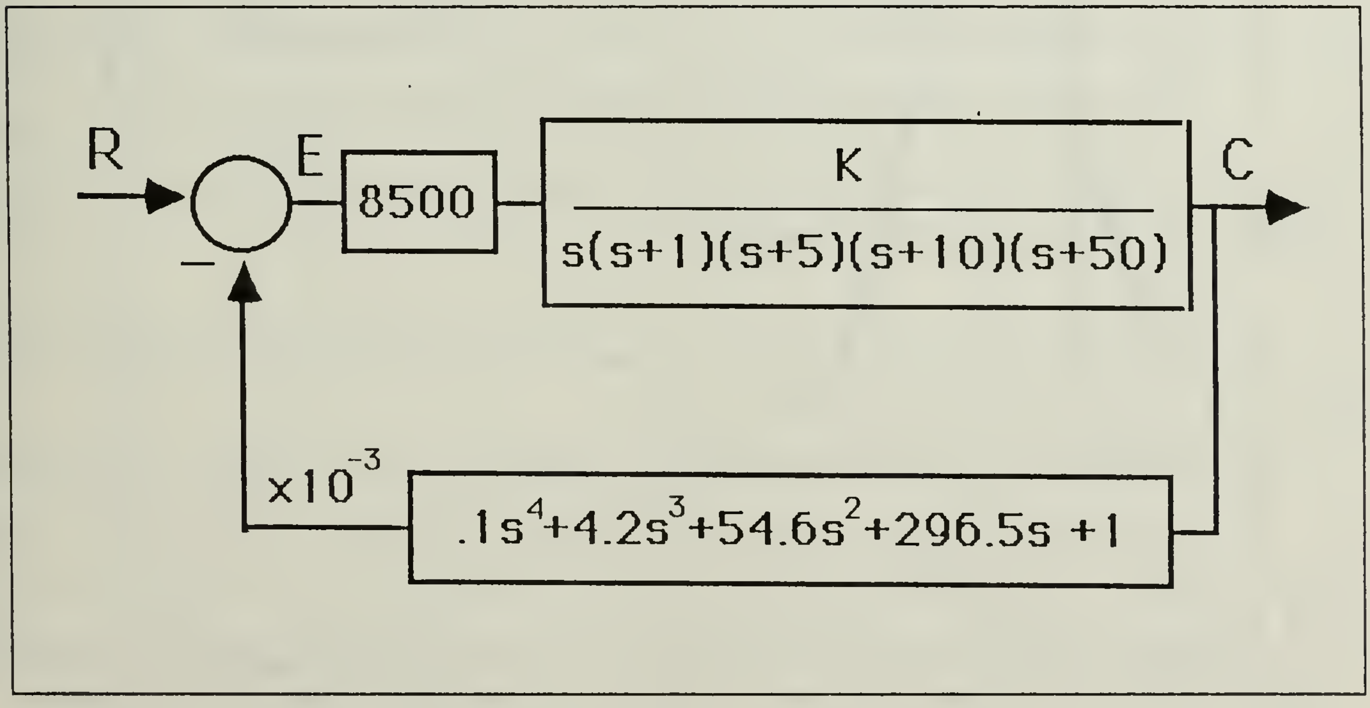

Figure 23. Unity Feedback Preservation

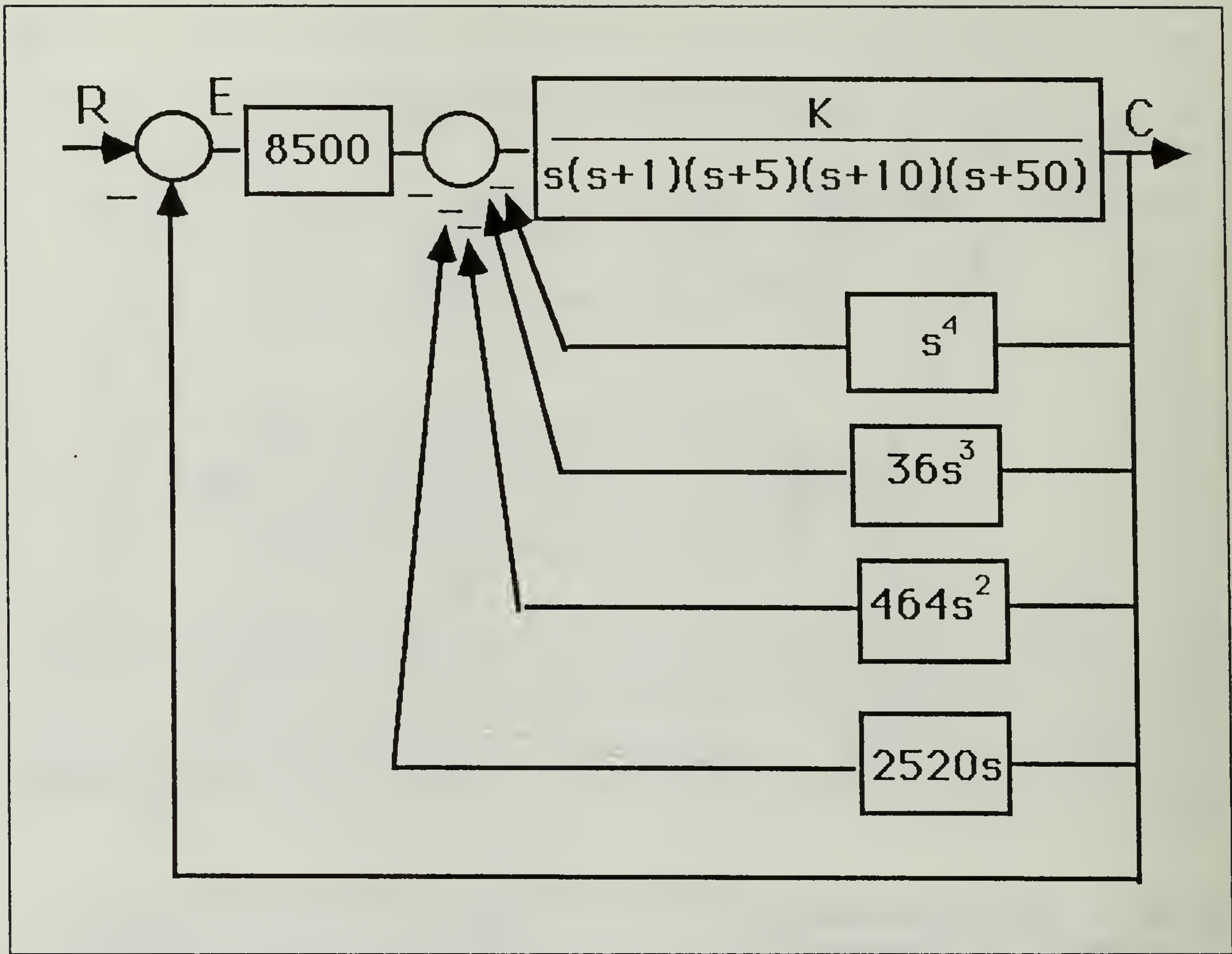

Figure 24. Rearranged Block Diagram

The root locii of the uncompensated and compensated systems are given in Figure 25 on page 29 and Figure 26 on page 30.

The compensated system open loop BODE diagram is given in Figure 27 on page 31. For the compensated system the Gain Crossover Frequency is 3.33 rad/sec, the Phase Margin is 46.64 Degrees, the Phase Crossover Frequency is 8.41 rad/sec, and the Gain Margin is 10.26 dB.The closed loop BODE diagram for the compensated system is given in Figure 28 on page 32.

The step response of the compensated system is given in Figure 29 on page 33. The system has a maximum overshoot to a step input at 0.8 sec exceeding the desired steady state value by about 14 % and has about 1.5 sec settling time.

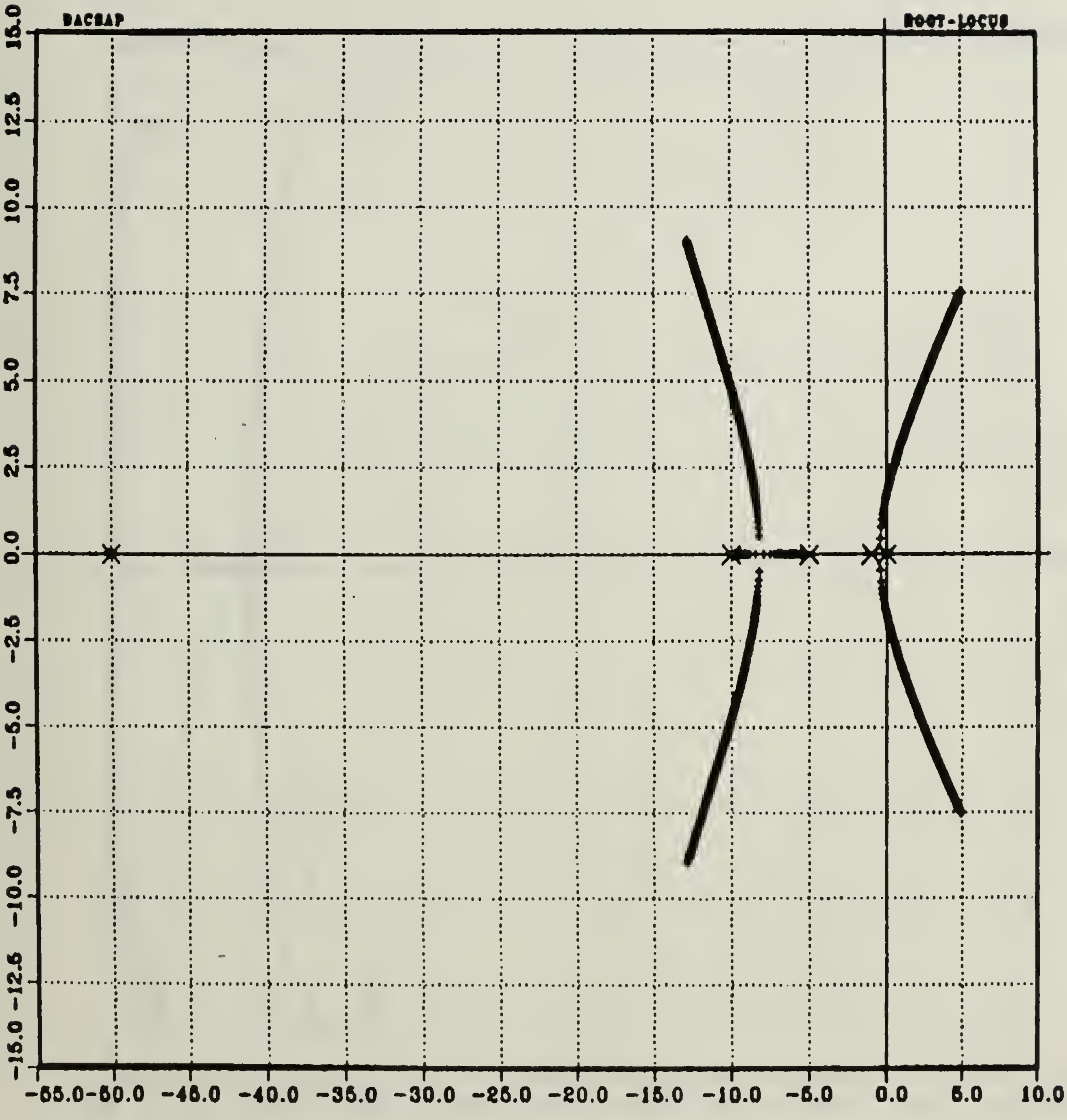

Figure 25. Root Locus of the Uncompensated System

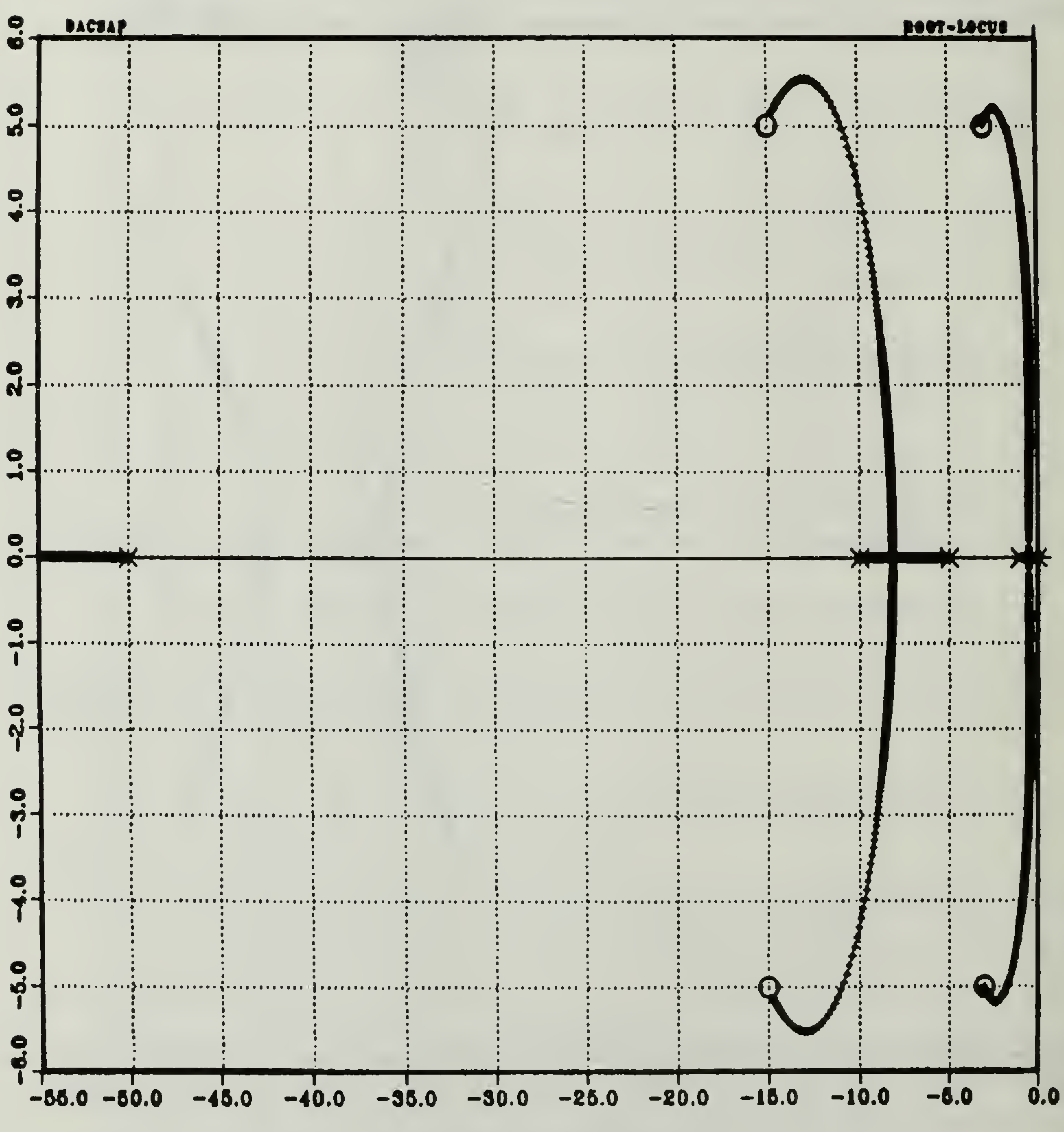

Figure 26. Root Locus of the Compensated System

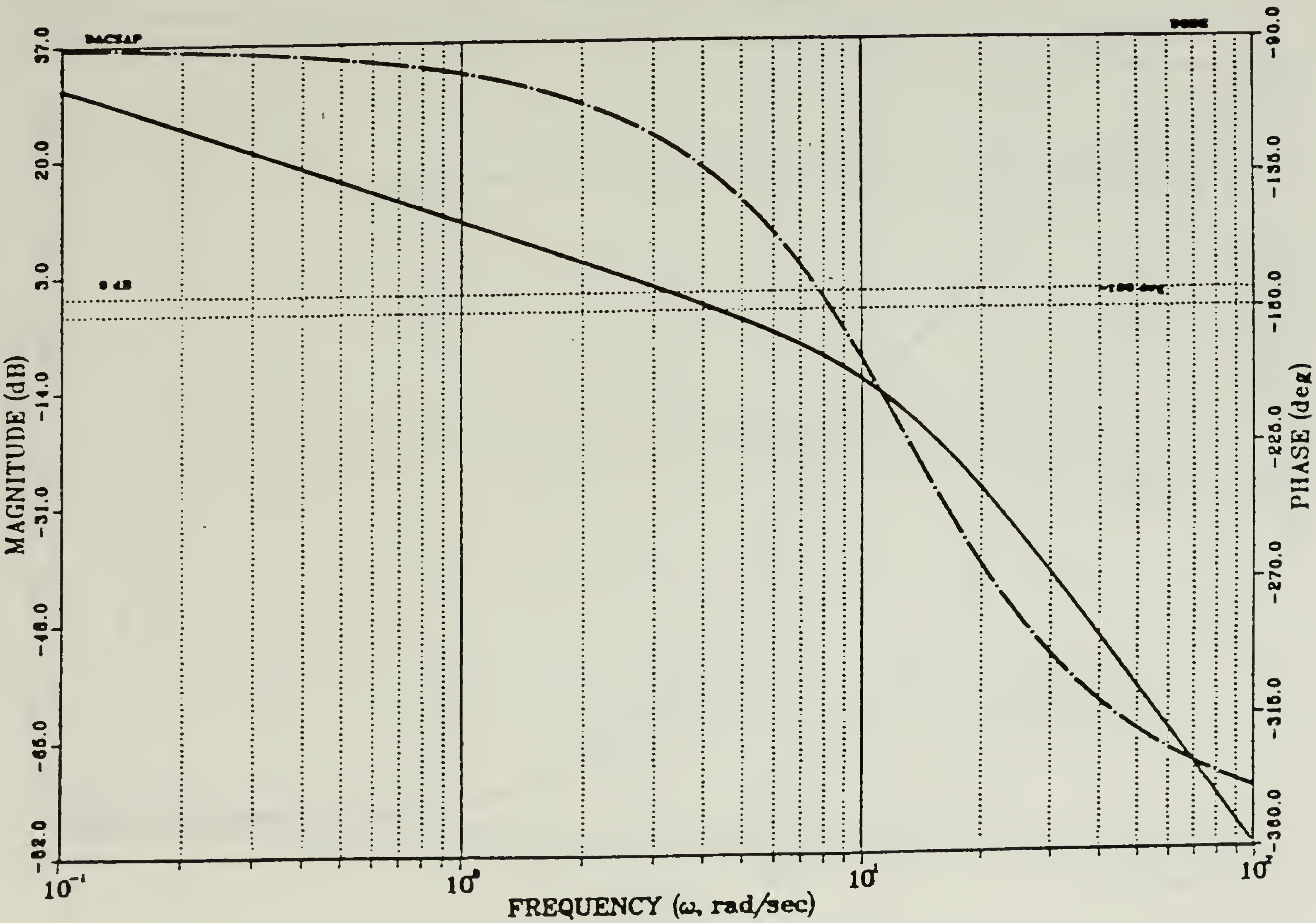

Figure 27. Compensated System Open Loop BODE Diagram

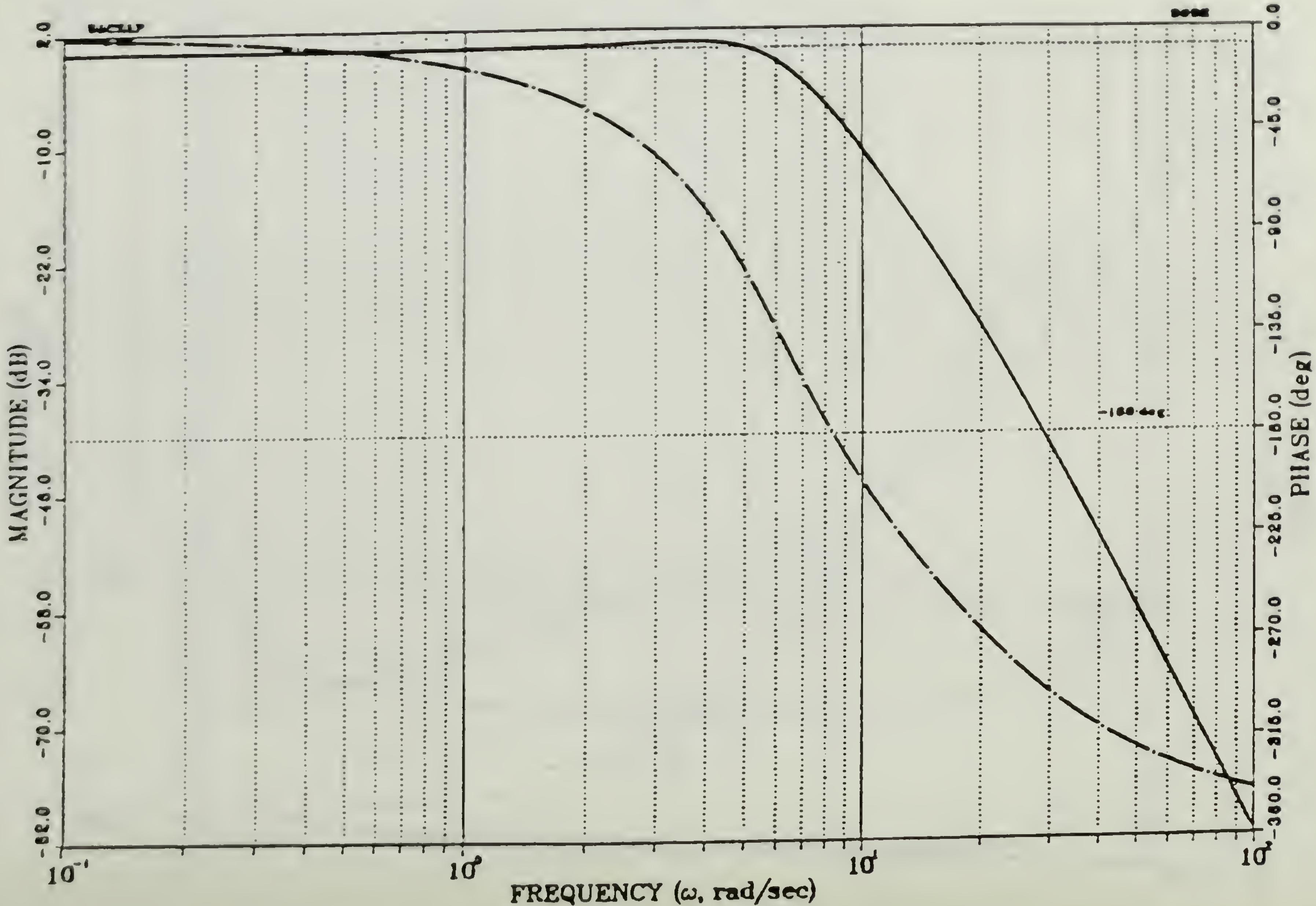

Figure 28. Compensated System Closed Loop BODE Diagram

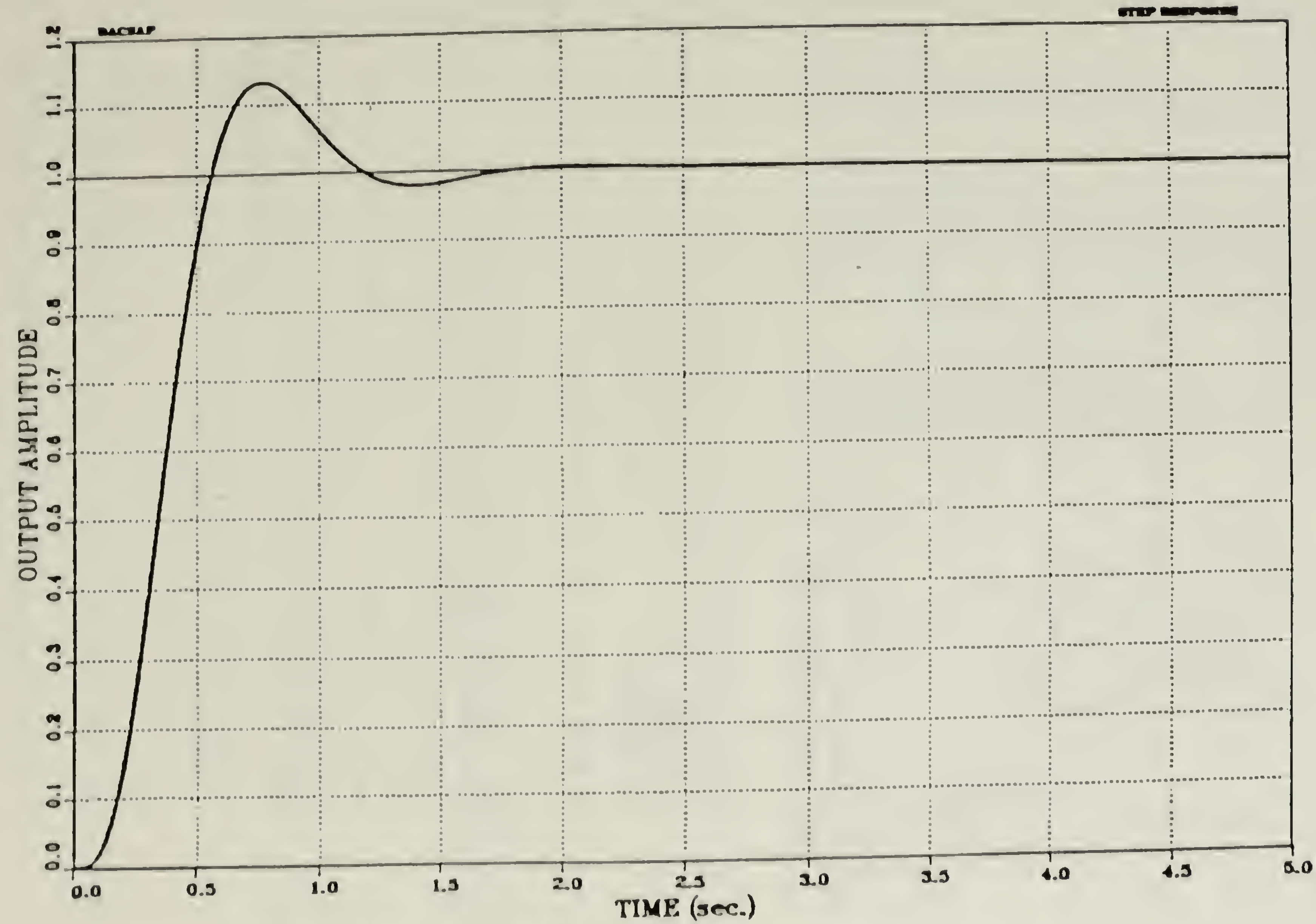

Figure 29. Step Response of the Compensated System

D. PLANT 4

The plant transfer function is:

$$G(s) = \frac{K}{(s+2)(s+30)(s^2+2s+100)} \tag{2.32}$$

The order of the plant is four (i.e., N = 4).

Name (N-1) roots:

Desired roots chosen;

-10.000 ± j5.000

-25.000

So,

$$\begin{aligned} H(s) &= (s+10+j5)(s+10-j5)(s+25) \\ &= s^3 + 45s^2 + 625s + 3125 \end{aligned} \tag{2.33}$$

The closed loop transfer function is:

$$G_{eq}(s) = \frac{G(s)}{1 + G(s)H(s)} \tag{2.34}$$

where,

$$G(s)H(s) = \frac{K}{(s+2)(s+30)(s^2+2s+100)} s^3 + 45s^2 + 625s + 3125 \tag{2.35}$$

The characteristic equation is:

$$1 + G(s)H(s) = 0 \tag{2.36}$$

$$1 + \frac{K(s^3 + 45s^2 + 625s + 3125)}{(s+2)(s+30)(s^2+2s+100)} = 0 \tag{2.37}$$

$$\text{CE: } s^4 + (34+K)s^3 + (224+45K)s^2 + (3320+625K)s + 6000 + 3125K = 0 \tag{2.38}$$

The locations of all roots as a function of block gain, K, are given in Table 4 on page 35.

Table 4. THE LOCATIONS OF ROOTS AS A FUNCTION OF BLOCK GAIN, K

$s^4 + (34 + K)s^3 + (224 + 45K)s^2 + (3320 + 625K)s + 3125K + 6000 = 0$				
K	r_1	r_2	r_3	r_4
1	-30.32	-1.1406	±j 10.6649	-2.6367
5	-30.68	-2.1922	±j 12.8561	-4.1771
10	-30.95	-3.8924	±j 14.6346	-5.2459
50	-43.30	-16.0474	±j 13.3540	-8.5959
100	-88.31	-16.9451	±j 5.0459	-11.5027
500	-488.89	-24.2876	-10.4458	±j 4.7953
1000	-988.43	-24.3267	-10.2118	±j 4.9056
5000	-4988.98	-24.9371	-10.0408	±j 4.9822
10000	-9988.99	-24.9687	-10.0203	±j 4.9912
50000	-49988.99	-24.9938	-10.0040	±j 4.9983
100000	-99988.99	-24.9968	-10.0020	±j 4.9991

Table 4 shows the gain required to move the specified roots to their desired locations and the location of the fourth unspecified root. When the block gain, K, equals 5000, the error is about 1% which is acceptable.

The system block diagram with state feedback and required block gain, K is given in Figure 30 on page 36.

If we rearrange the gains to preserve unity feedback, the feedback plant becomes:

$$H(s) = s^3 + 45s^2 + 625s + 3125 \tag{2.39}$$

$$H(s) = 3125(0.00032s^3 + 0.0144s^2 + 0.2s + 1) \tag{2.40}$$

In Figure 31 on page 36, the unity feedback is obtained for the system by adjusting the forward and feedback gains.

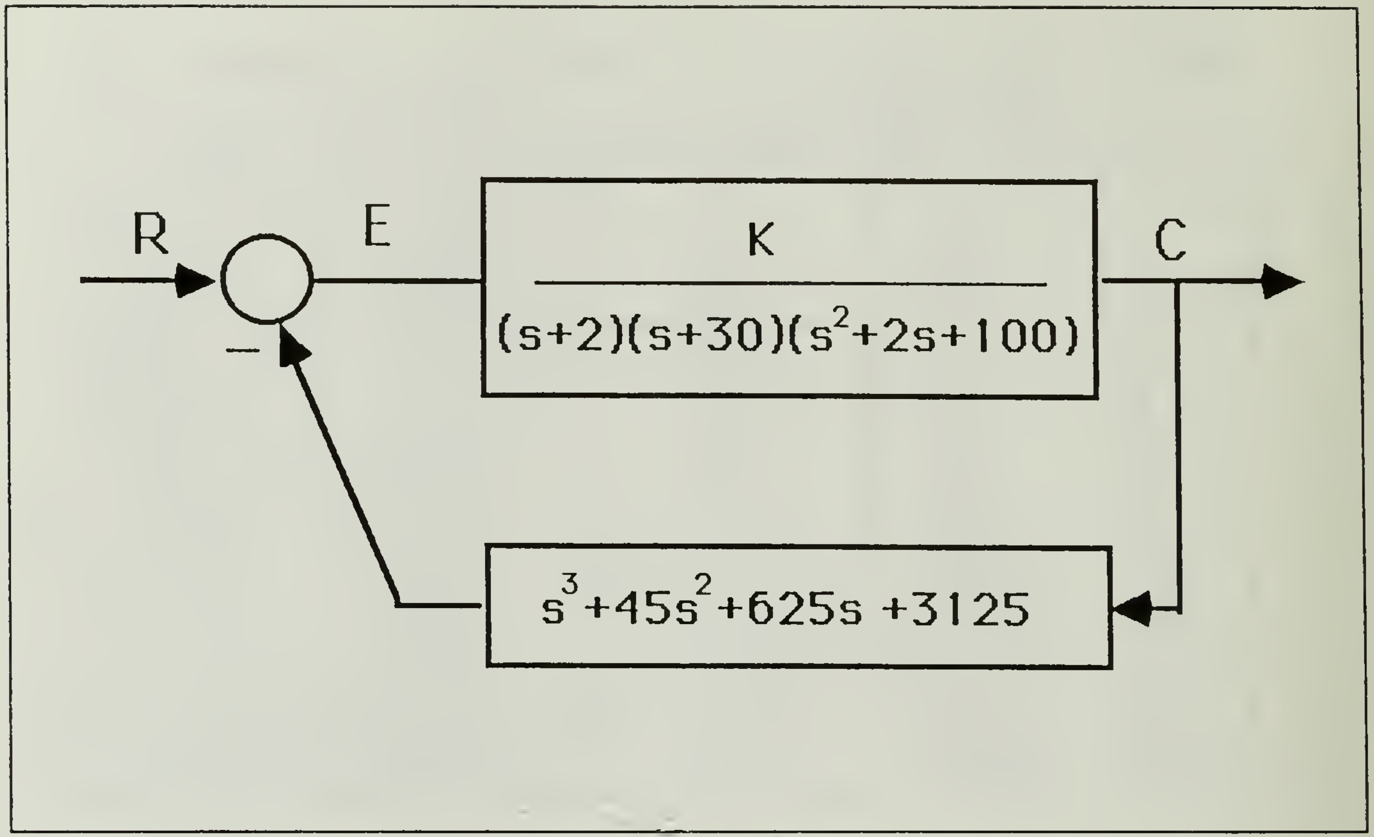

Figure 30. Basic State Feedback Block Diagram

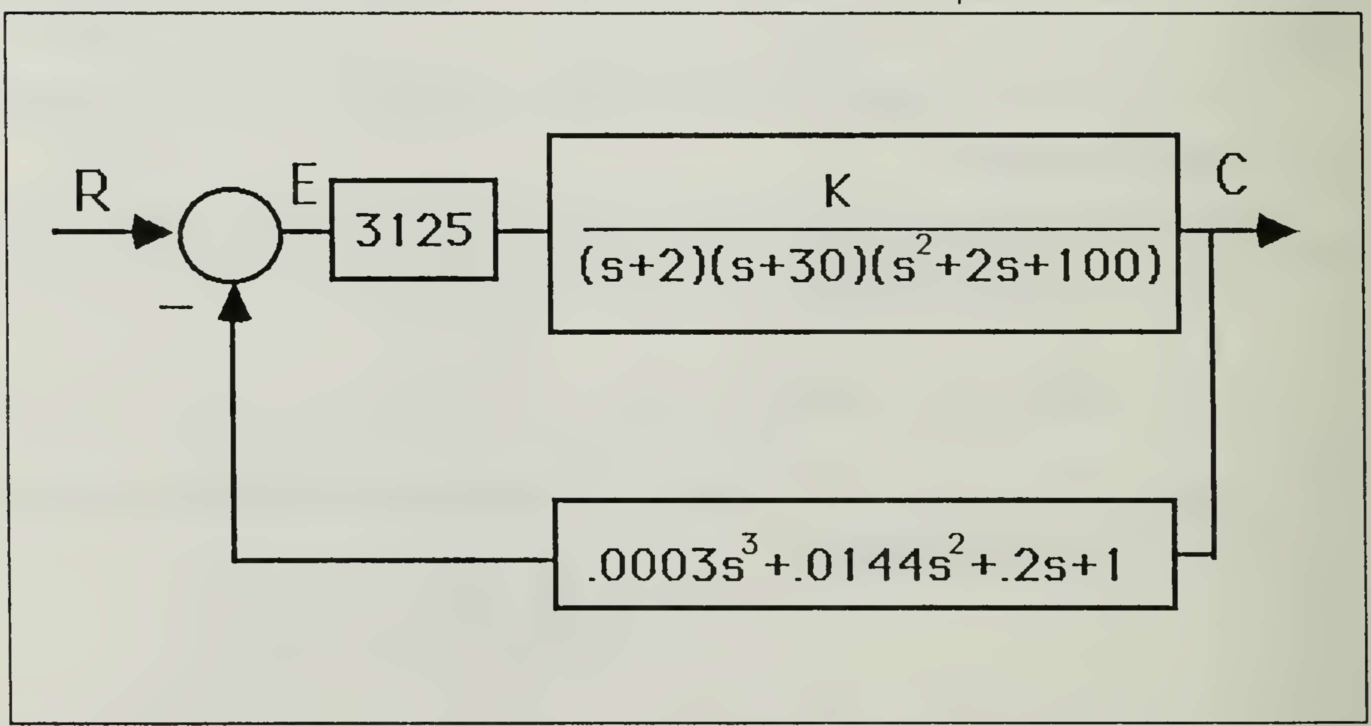

Figure 31. Unity Feedback Preservation

The rearranged block diagram is given in Figure 32 on page 37.

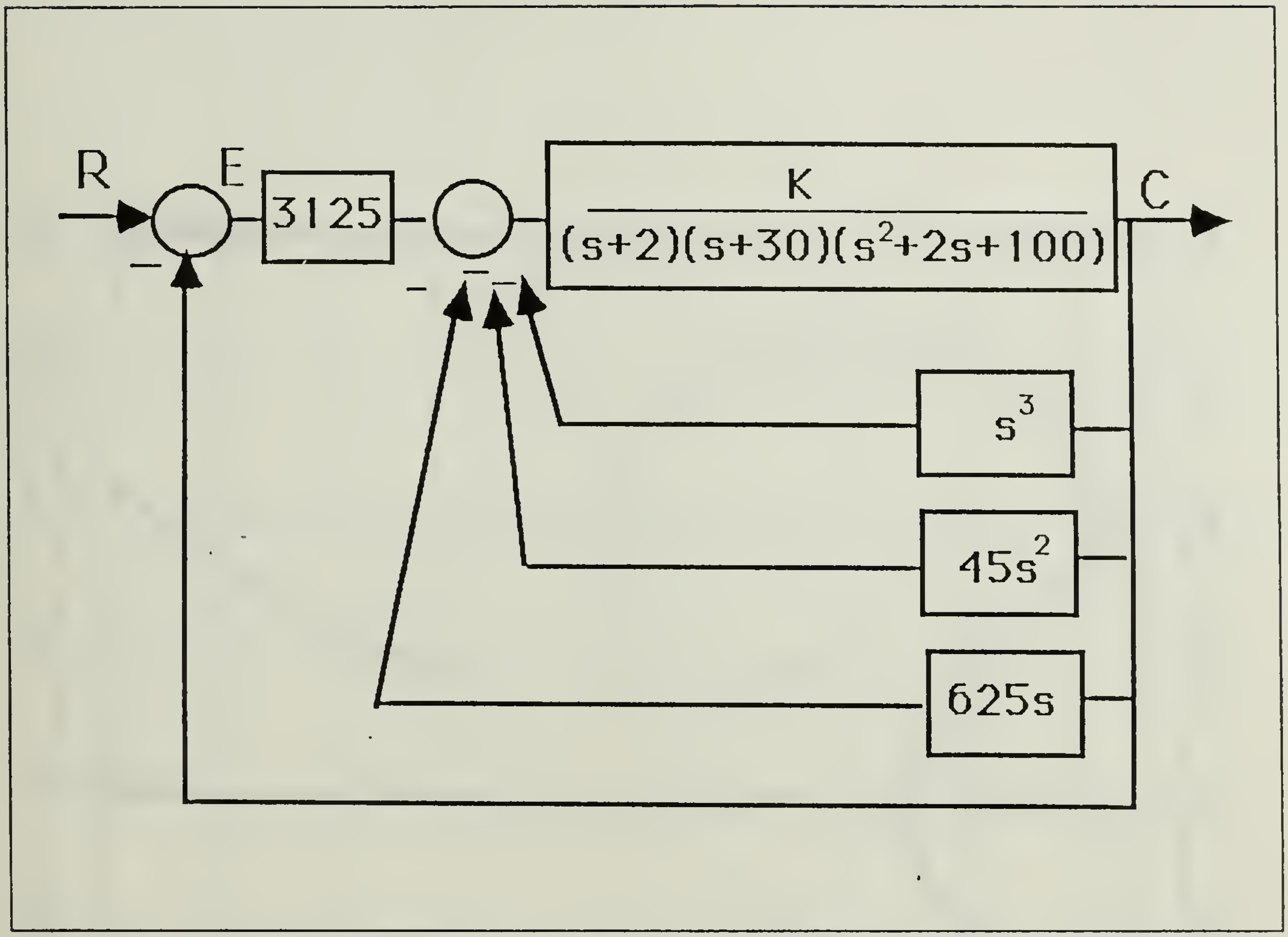

Figure 32. Rearranged Block Diagram

The root locii of the uncompensated and compensated systems are given in Figure 33 on page 38 and Figure 34 on page 39.

The compensated system open loop BODE diagram is given in Figure 35 on page 40. For the compensated system the Gain Crossover Frequency is 4.917 rad/sec, the Phase Margin is 69.78 Degrees, the Phase Crossover Frequency is 25.03 rad/sec, and the Gain Margin is 19.12 dB.

The closed loop BODE diagram for the compensated system is given in Figure 36 on page 41.

The step response of the compensated system is given in Figure 37 on page 42. The system does not have an overshoot to a step input and becomes stable at 0.5 sec.

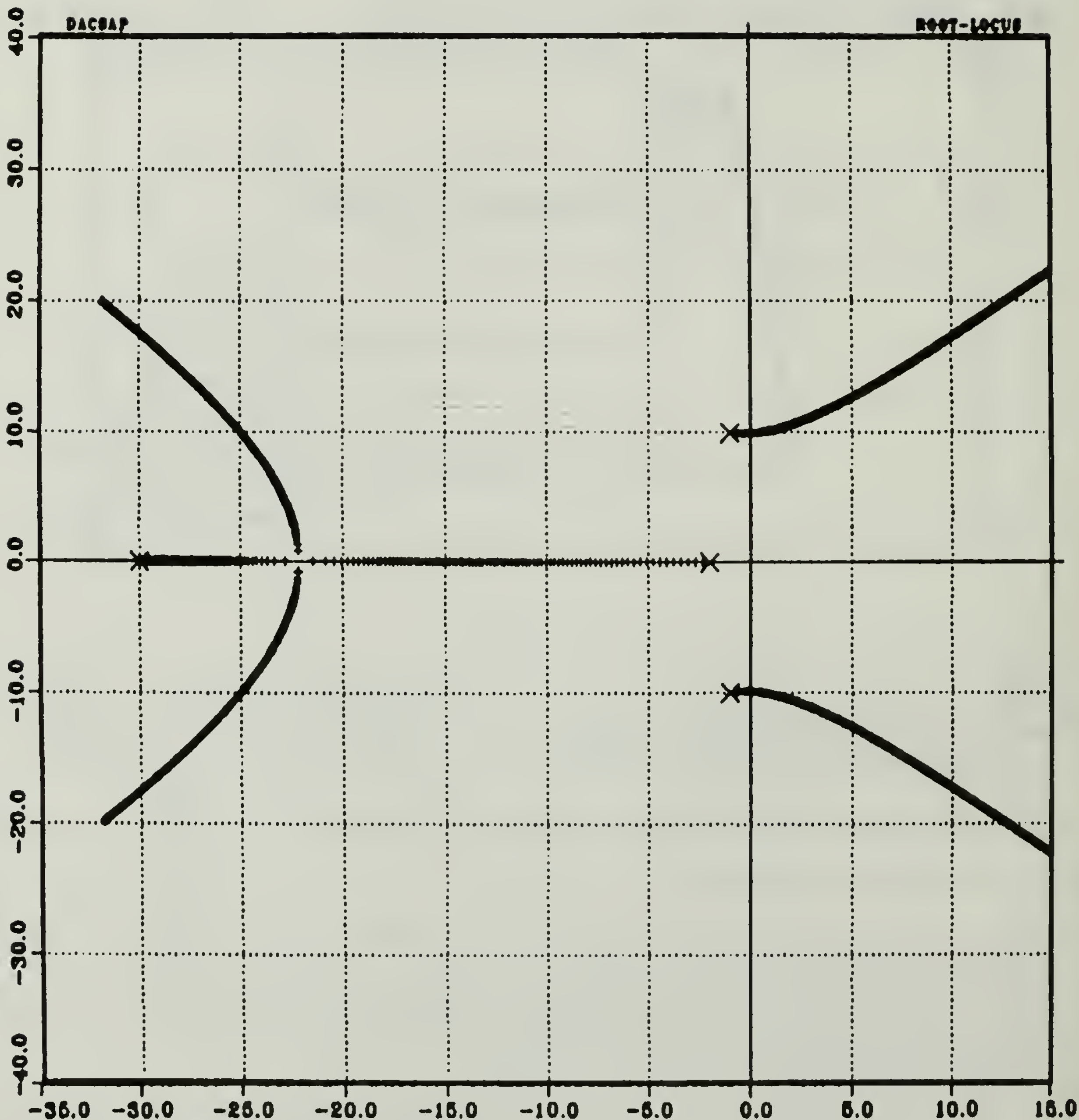

Figure 33. Root Locus of the Uncompensated System

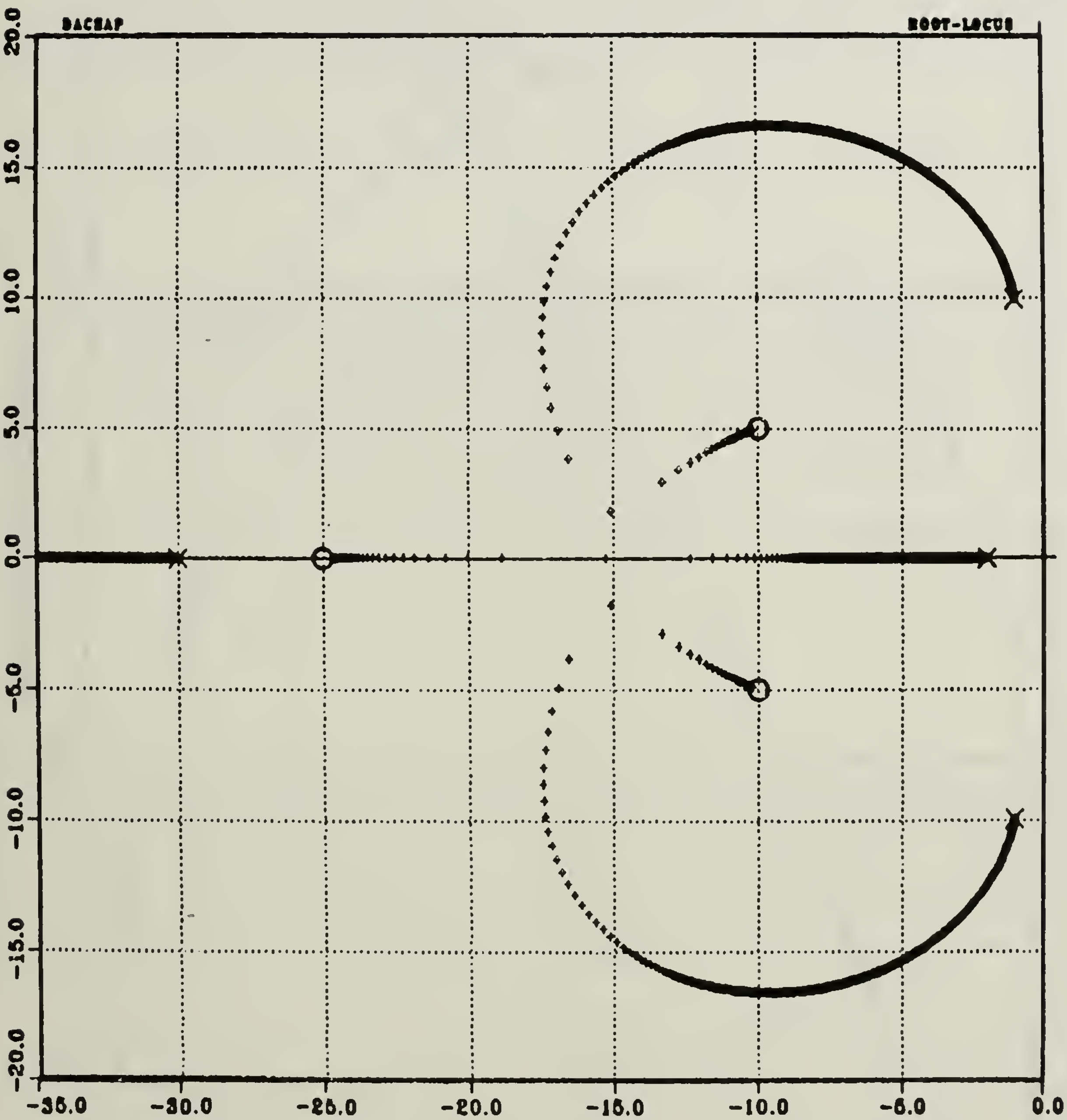

Figure 34. Root Locus of the Compensated System

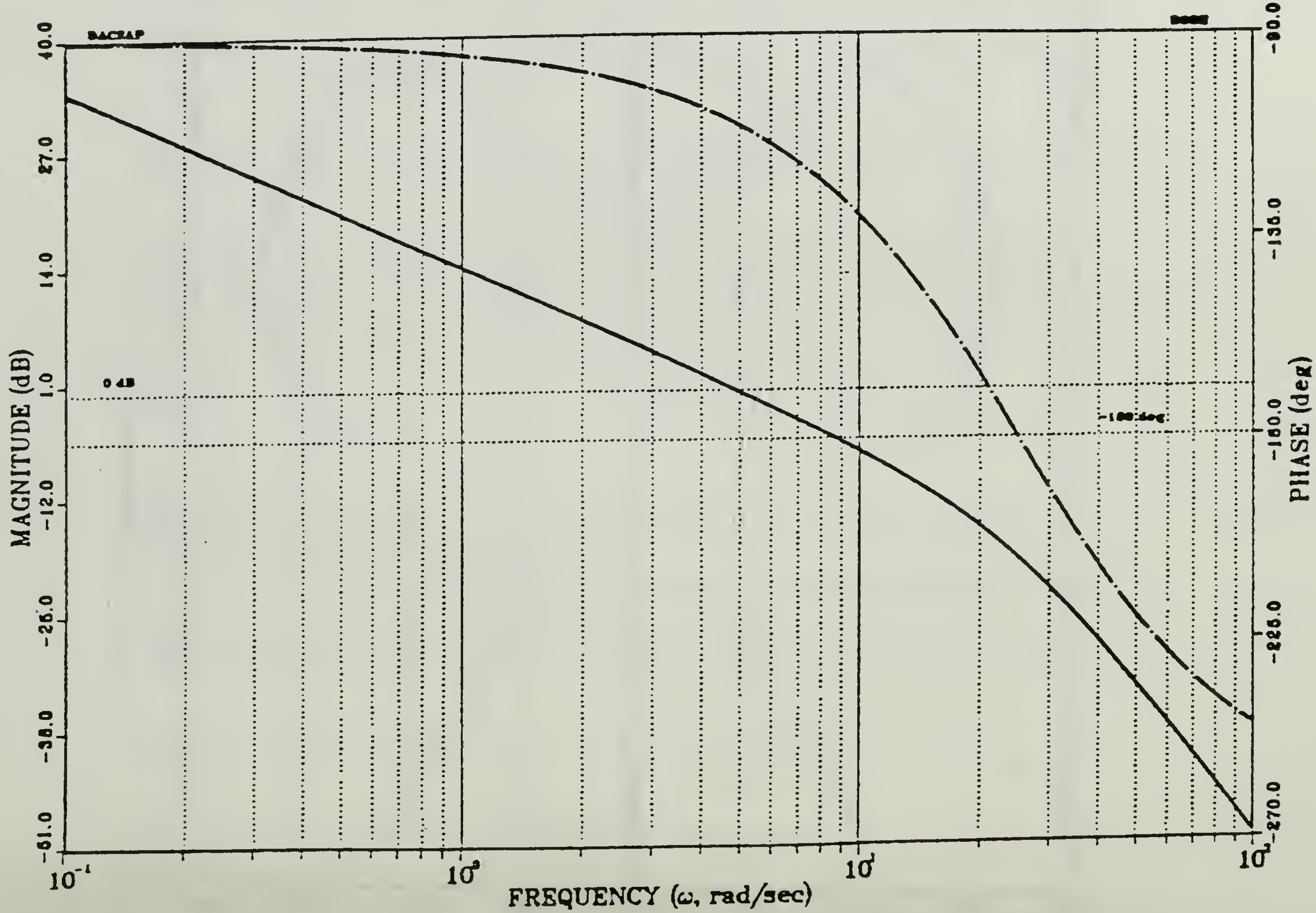

Figure 35. Compensated System Open Loop BODE Diagram

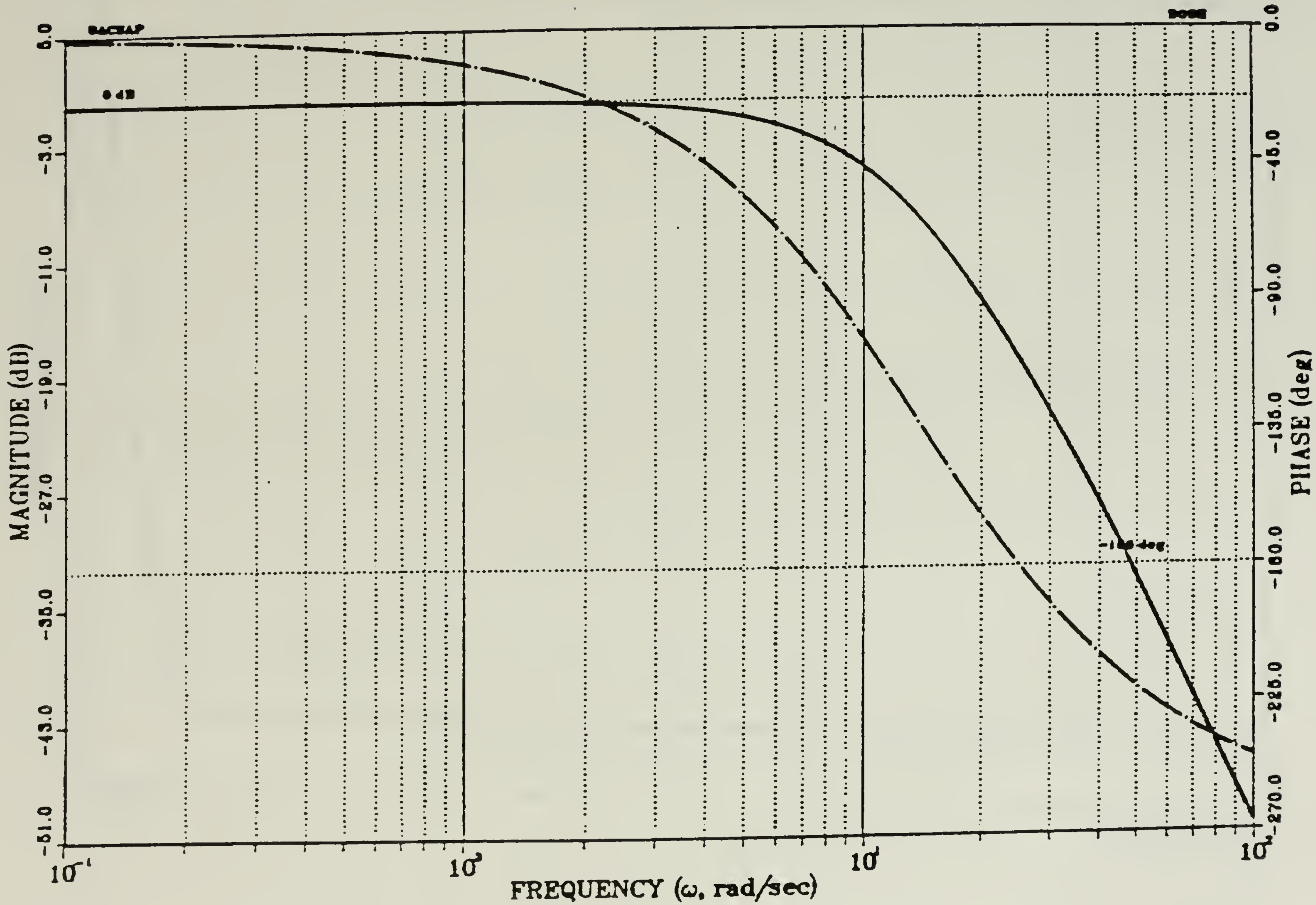

Figure 36. Compensated System Closed Loop BODE Diagram

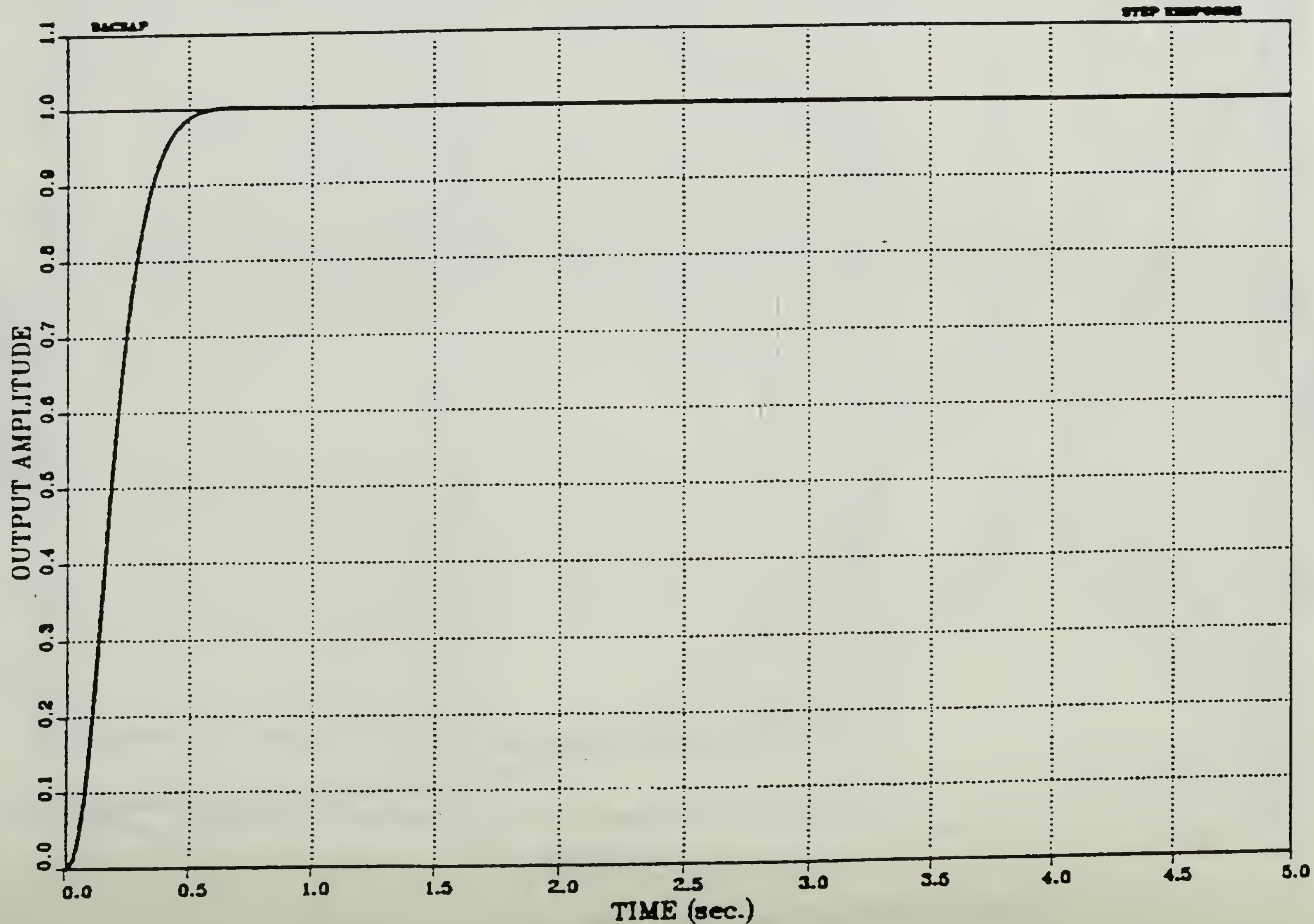

Figure 37. Step Response of the Compensated System

E. CONCLUSIONS

In using the transfer function method to place roots at the desired locations, the roots move to the desired location very fast for small increses of the gain. The movement of the roots becames slower when the roots are close to zeros. Then, for large increases of the gain, the root movement is very slow. So, large gain is not required to obtain acceptable root location. On the other hand, the unspecified root follows on negative part of the real axis towards infinity as the gain of the system approaches infinity.

III. DESIGN BY MATRIX METHODS AND OBSERVATION OF OFFSETTING THE ZEROS

The SVS pole placement program requires N roots to be specified and outputs the equation for H(s) based on the input roots and the value for the forward path gain, K, required to place the poles as specified. It must be noted that the data output is for the case where unity feedback is preserved therefore the value for K output by SVS must be divided by k_0 to be compared with the values of K used in the transfer function method.

Matrix methods located the roots at the desired locations by adjusting the zeros to the offset locations. How the zeros are adjusted by matrix methods will be observed by studying a number of cases of all-pole plants.

A. DESIGN BY MATRIX METHODS

A plant for a third order system is defined

$$G(s) = \frac{K}{s^3 + 3s^2 + 2s} \tag{3.1}$$

Desired roots:

-1000

-1.000 ± j 2.000

and K = 300 are specified. Then

$$G(s) = \frac{C(s)}{R(s)} = \frac{Y(s)}{U(s)} = \frac{300}{s^3 + 3s^2 + 2s} \tag{3.2}$$

$$\dddot{y} + 3y + 2\dot{y} = u \tag{3.3}$$

$$\dddot{y} = u - 3y - 2\dot{y} \tag{3.4}$$

$$\dot{y} = Ay + Bu \tag{3.5}$$

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 300 \end{bmatrix} u \tag{3.6}$$

The plant matrix A is:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} \tag{3.7}$$

The input matrix B is:

$$B = \begin{bmatrix} 0 \\ 0 \\ 300 \end{bmatrix} \tag{3.8}$$

The output matrix C is:

$$C = [1 \quad 0 \quad 0] \tag{3.9}$$

Desired closed-loop characteristic polynomial obtained from the desired roots is:

$$(s + 1000)(s + 1 + j2)(s + 1 - j2) = s^3 + 1002s^2 + 2005s + 5000 \tag{3.10}$$

When we enter the data above, the SVS results are:

Feedback coefficients:

$$k = [16.6667 \quad 6.6767 \quad 3.3300] \tag{3.11}$$

The closed loop gain:

K = 1.0000

$$\begin{aligned} H(s) &= 3.3300s^2 + 6.6767s + 16.6667 \\ &= 3.3300(s^2 + 2.0050s + 5.00050) \end{aligned} \tag{3.12}$$

The H(s) quadratic is factored to see location of zeros:

-1.00250751 + j1.99999843

-1.00250751 -j1.99999843

These results show that the matrix methods move the zeros from -1.000 ∓ j2.000 to -1.00250751 ∓ 1.99999843. Therefore the SVS program relocated zeros at the places which are a little farther away from the desired locations. A detailed analysis on plant 1 will be done in part B.

B. OBSERVATION OF OFFSETTING THE ZEROS BY MATRIX METHODS

In order to find some guidance to determine in which direction and how far the offset-zero locations must be located, a number of cases are studied considering only all-pole plants. To do this we select a number of values for the desired N-th root without changing the values of the other N-1 roots. The SVS program determines the required gain for each case.

1. Plant 1

Transfer function:

$$G(s) = \frac{K}{s(s+1)(s+2)} \tag{3.13}$$

As seen in the plot of the forward and feedback gains (Figure 38 on page 47), the gains are decreased as the N-th root moves close to the origin. During this process, the poles of the plant are always at the same locations, but the zeros of the system are changing. When the N-th root location is far away (at infinity), the zeros are at the desired location. The zeros go away from the desired location as the gains decrease (Figure 39 on page 48).

To observe the offset zero locations a number of different desired roots which are shown below are studied using plant 1.

The desired roots are:

1. $-1.000 \mp j2.000$ (Figure 40 on page 51 - Figure 43 on page 54)
2. $-0.500 \mp j6.000$ (Figure 44 on page 55 - Figure 47 on page 58)
3. $-5.000 \mp j10.000$ (Figure 48 on page 59 - Figure 51 on page 62)

For each set of desired roots, the forward and feedback gains are found by changing the N-th root location.

The last study is the root locus curve for each case.

Table 5 on page 49 shows the output of the SVS program, k_0, k_1, and k_2. These are the coefficients of the feedback transfer function:

$$H(s) = k_2 s^2 + k_1 s + k_0 \tag{3.14}$$

In order to see location of the zeros, the H(s) quadratic is factored in Table 6 on page 50.

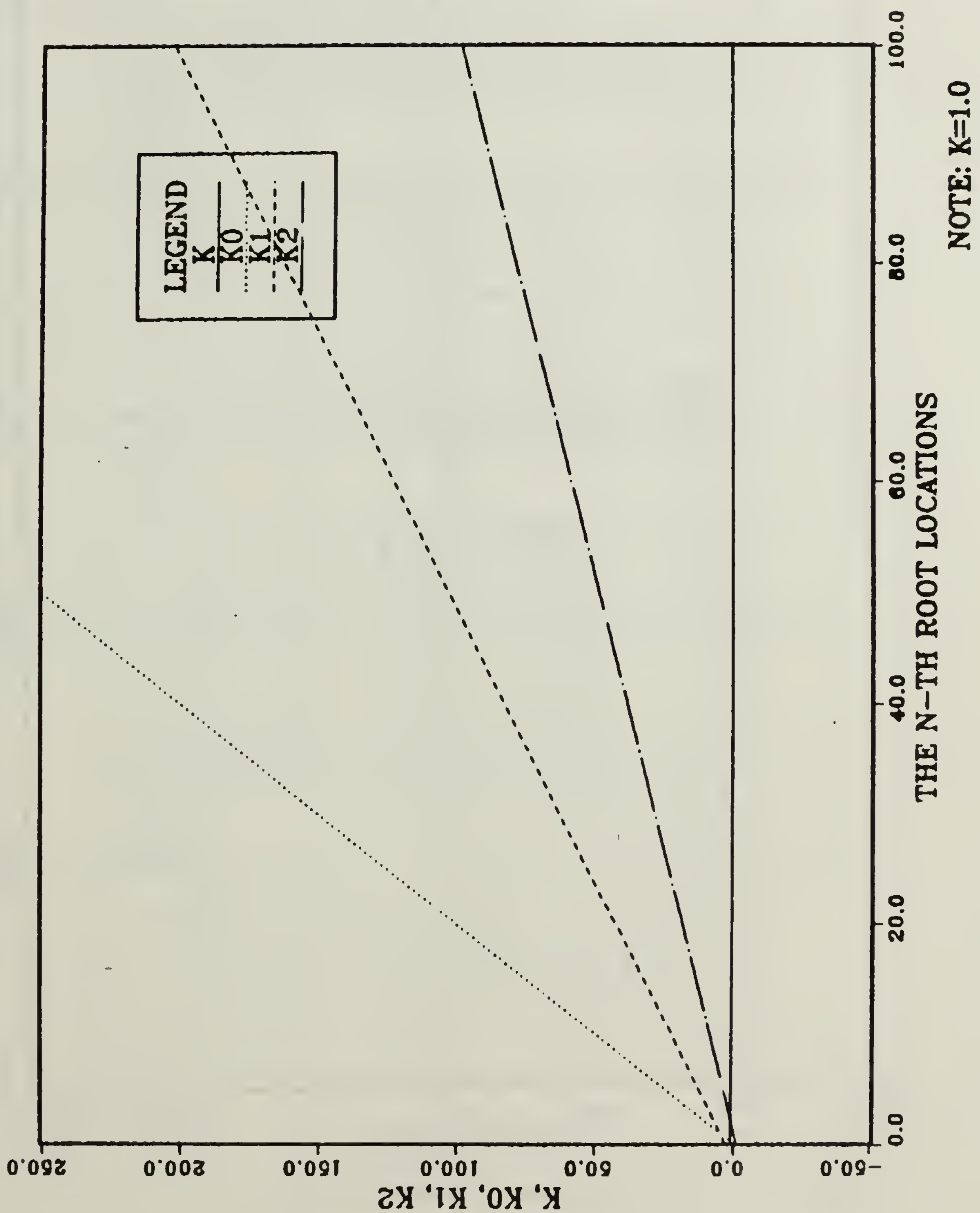

Figure 38. Forward and Feedback Gains As A Function Of The n-th Root Location

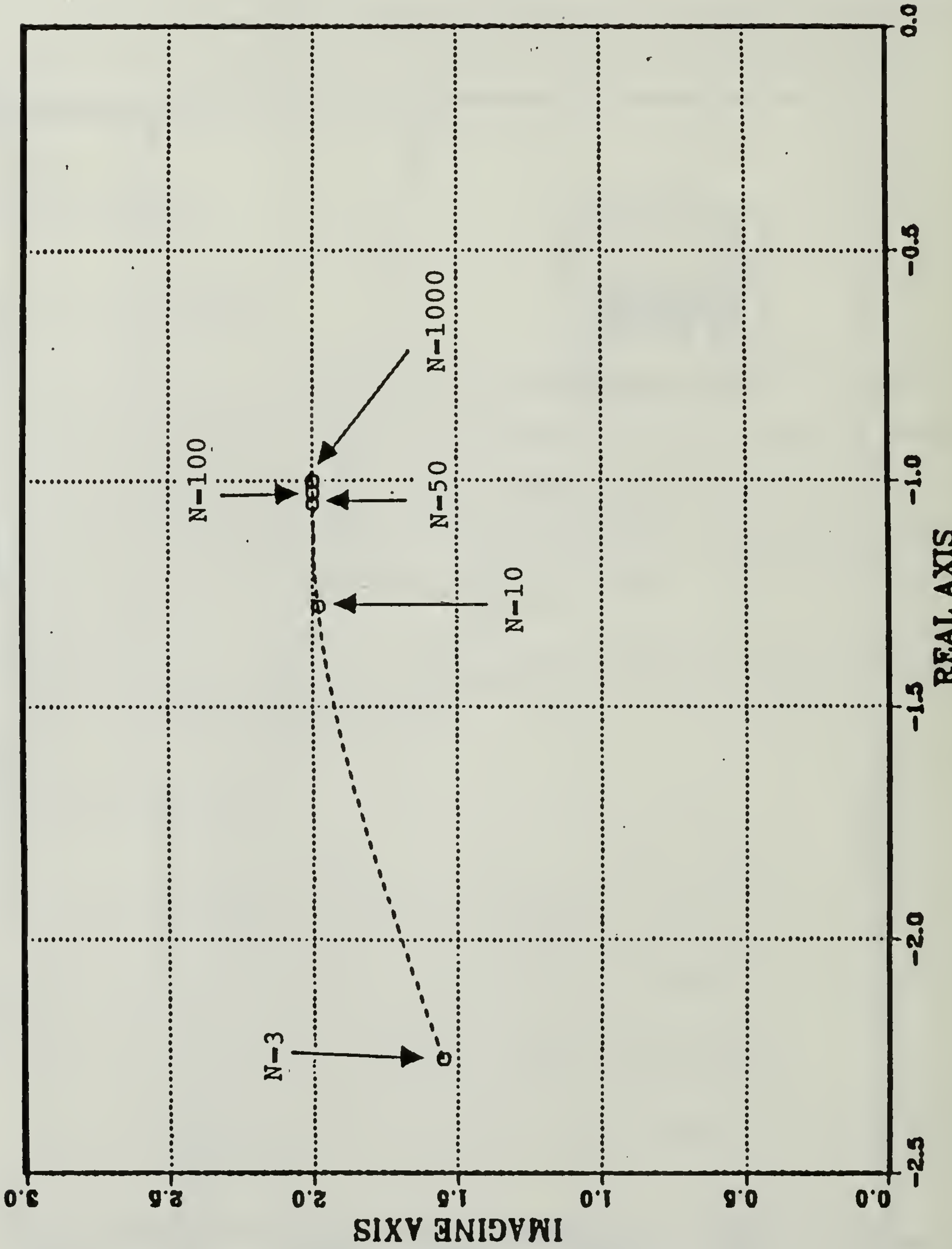

Figure 39. Offsetting Zeros by Matrix Methods

Table 5. THE FEEDBACK COEFFICIENTS AS A FUNCTION OF THE N-TH ROOT

$$G(s) = \frac{K}{s(s+1)(s+2)}$$

DESIRED ROOTS ; -1.000 ∓ j2.000			
LOCATION OF N-TH ROOTS	FEEDBACK COEFFICIENTS		
	k_0	k_1	k_2
-10000	50000	20003	9999
-1000	5000	2003	999
-100	500	203	99
-50	250	103	49
-20	100	43	19

DESIRED ROOTS ; -0.500 ∓ j6.000			
LOCATION OF N-TH ROOTS	FEEDBACK COEFFICIENTS		
	k_0	k_1	k_2
-10000	362500	10034	9998
-1000	36250	1034	998
-100	3625	134	98
-50	1812	84	48
-30	1087	64	28

DESIRED ROOTS ; -5.000 ∓ j10.000			
LOCATION OF N-TH ROOTS	FEEDBACK COEFFICIENTS		
	k_0	k_1	k_2
-10000	1250000	100123	10007
-1000	125000	10125	1007
-100	12500	1123	107
-50	6250	623	57
-30	3750	423	37

Table 6. OFFSET ZERO LOCATIONS BY MATRIX METHODS

$$G(s) = \frac{K}{s(s+1)(s+2)}$$

DESIRED ROOTS ; -1.000 ∓ j2.000		
LOCATION OF N-TH ROOTS	ZERO LOCATIONS	
	z_0	z_1
-10000	-1.00025	∓ j1.99999
-1000	-1.00250	∓ j1.99999
-100	-1.02525	∓ j1.99984
-50	-1.05102	∓ j1.99934
-20	-1.13157	∓ j1.99566

DESIRED ROOTS ; -0.500 ∓ j6.000		
LOCATION OF N-TH ROOTS	ZERO LOCATIONS	
	z_0	z_1
-10000	-0.50180	∓ j6.00045
-1000	-0.52327	∓ j6.03460
-100	-0.68357	∓ j6.04337
-50	-0.87500	∓ j6.08148
-30	-1.14285	∓ j6.12497

DESIRED ROOTS ; -5.000 ∓ j10.000		
LOCATION OF N-TH ROOTS	ZERO LOCATIONS	
	z_0	z_1
-10000	-5.00265	∓ j9.99430
-1000	-5.02730	∓ j9.94269
-100	-5.24766	∓ j9.44907
-50	-5.46491	∓ j8.93218
-30	-5.71621	∓ j8.28711

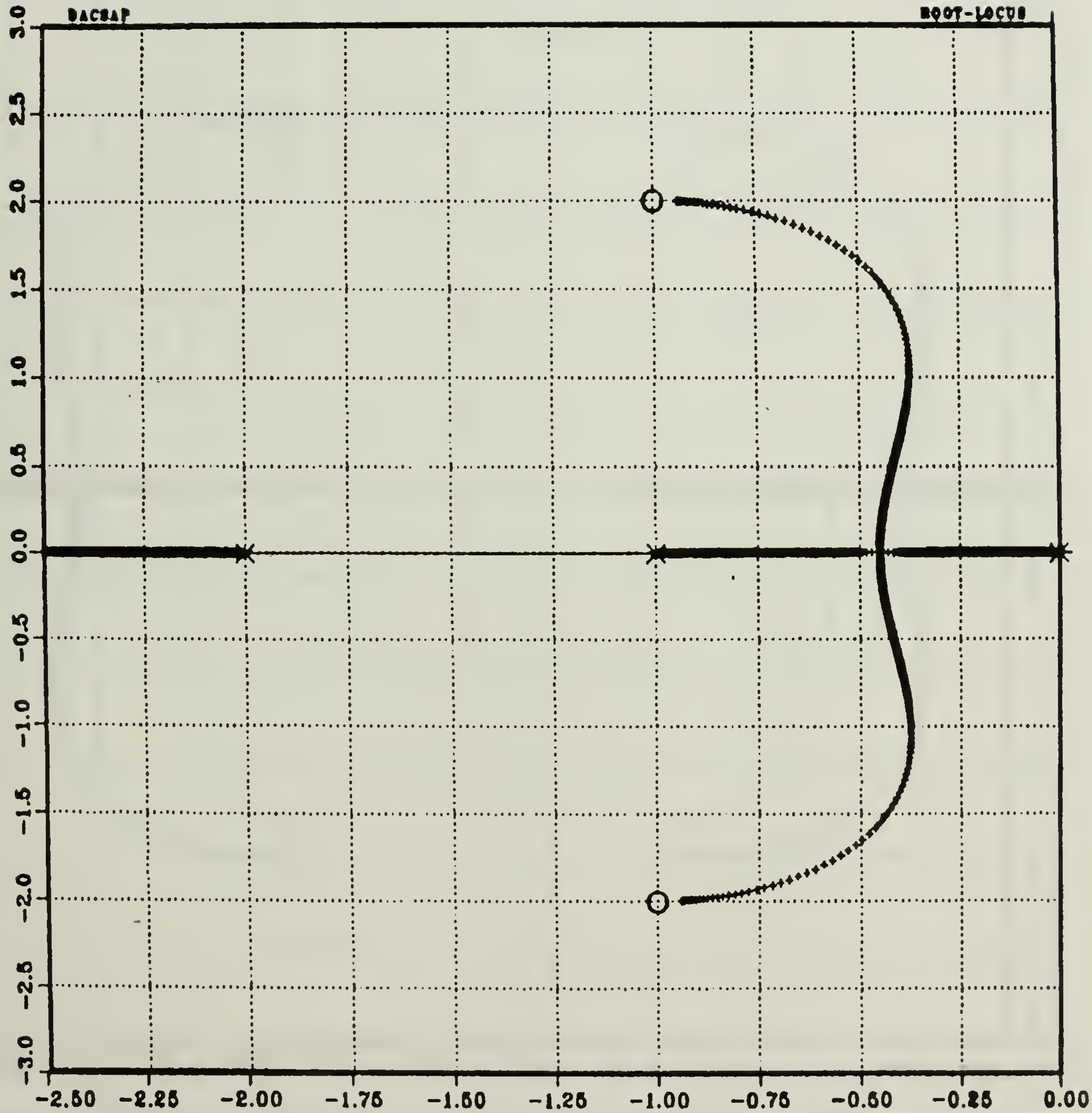

Figure 40. Compensated System Root Loci. Zeros at Location of Desired Roots; -1.000 ∓ j2.000

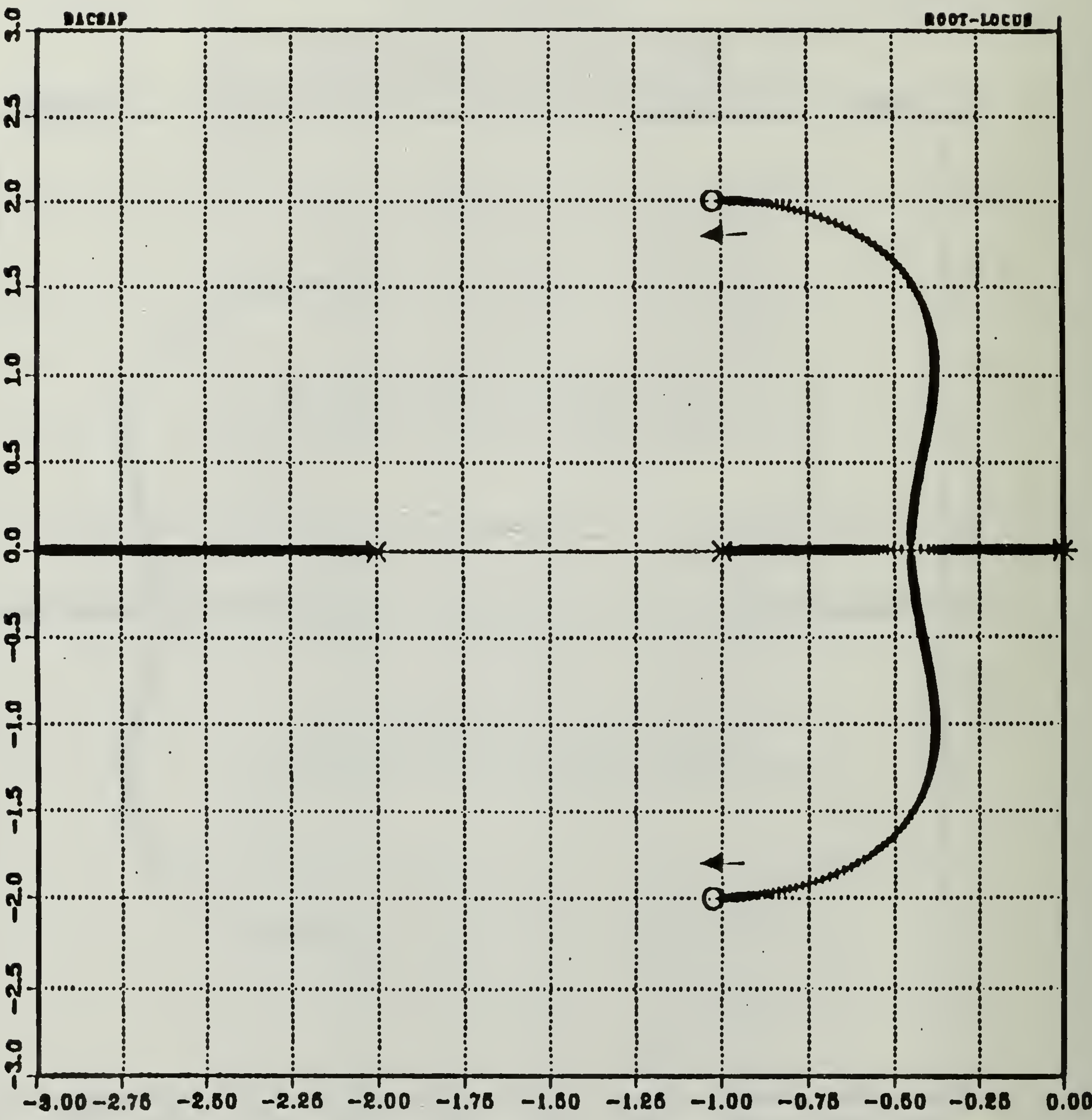

Figure 41. Offsetting Zeros by Matrix Methods. N-th Root at -100. Desire Roots; -1.000 ∓ j2.000

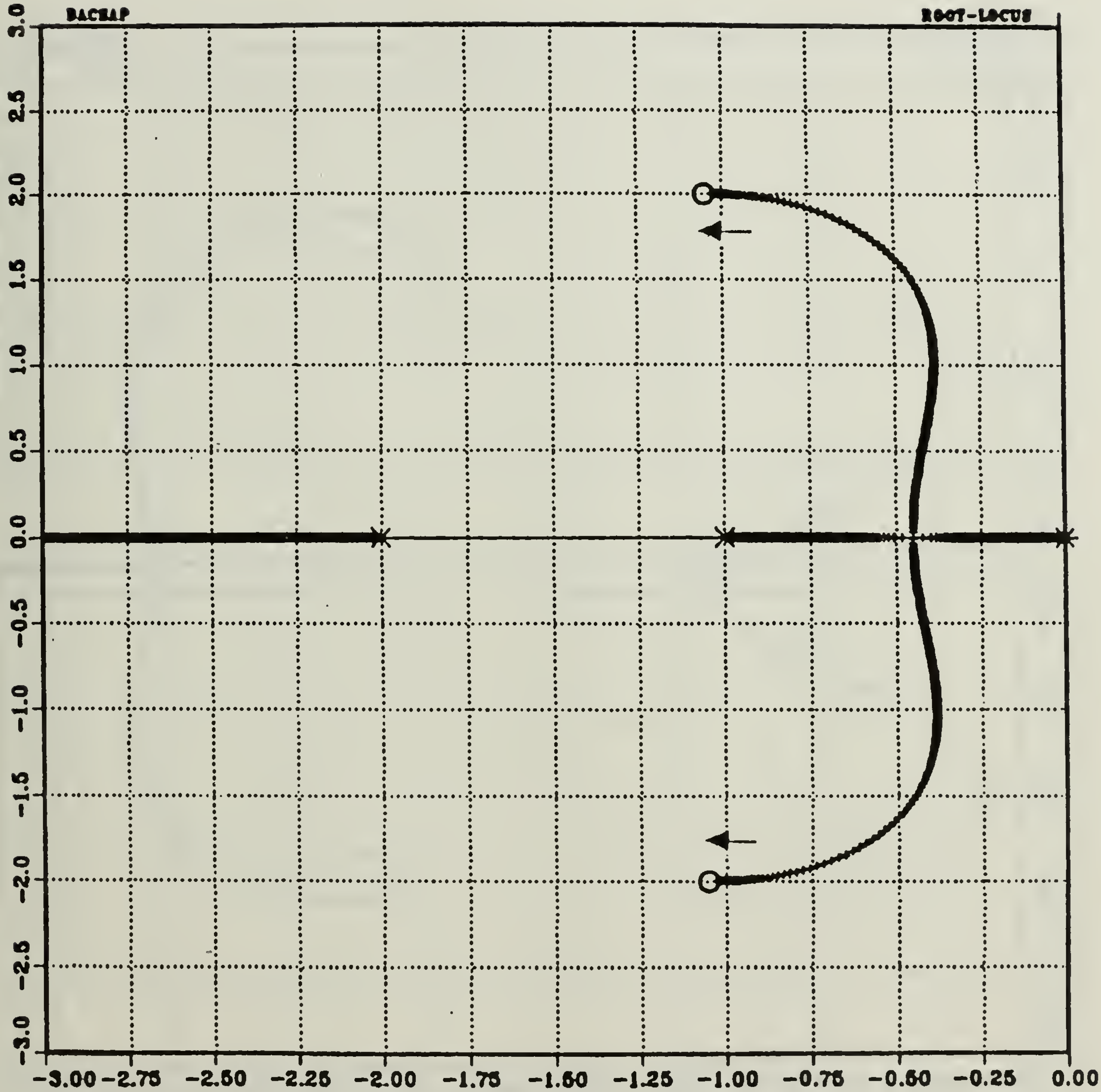

Figure 42. Offsetting Zeros by Matrix Methods. N-th Root at -50. Desire Roots; -1.000 ∓ j2.000

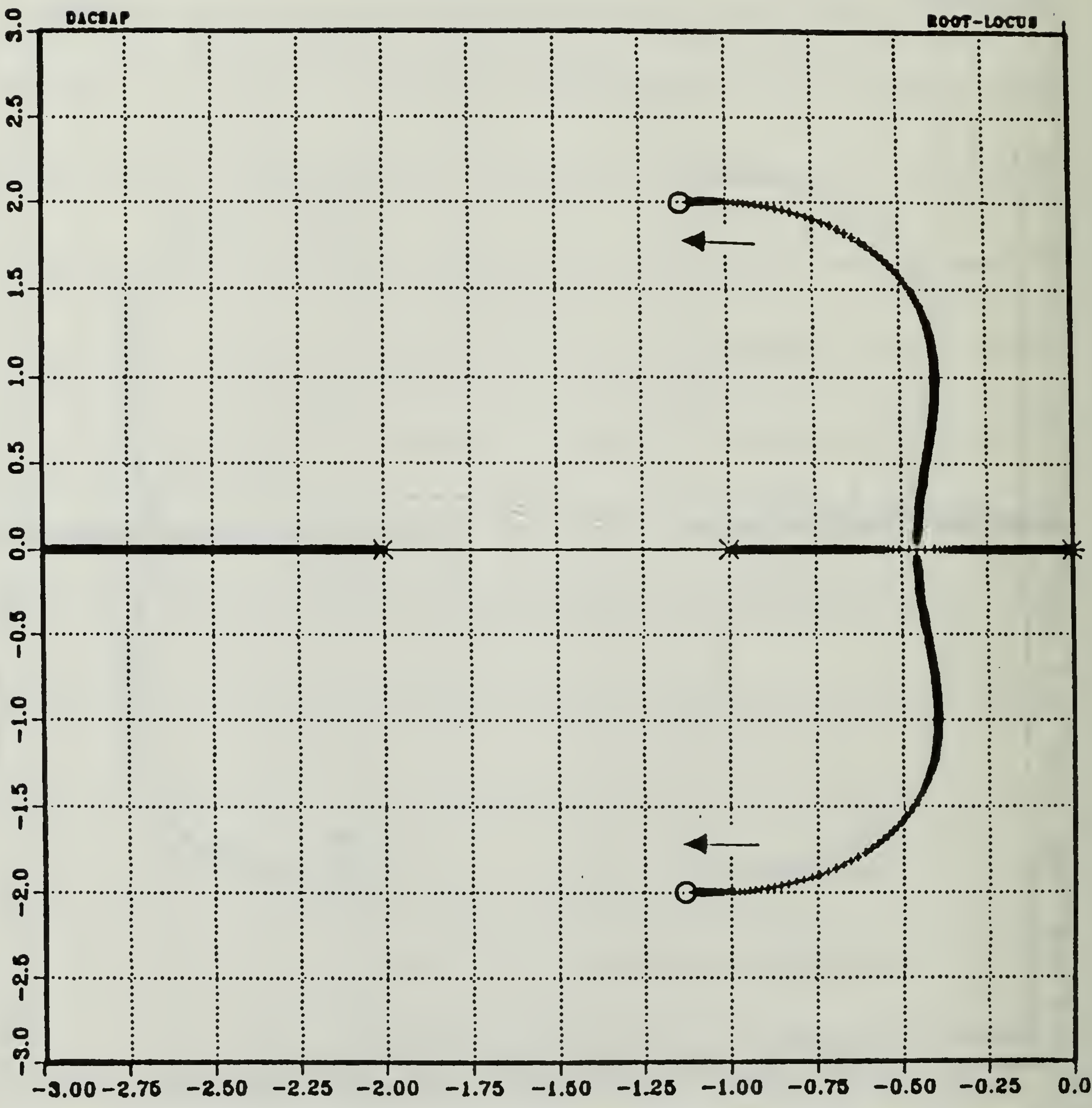

Figure 43. Offsetting Zeros by Matrix Methods. N-th Root at -20. Desired Roots; -1.000 ∓ j2.000

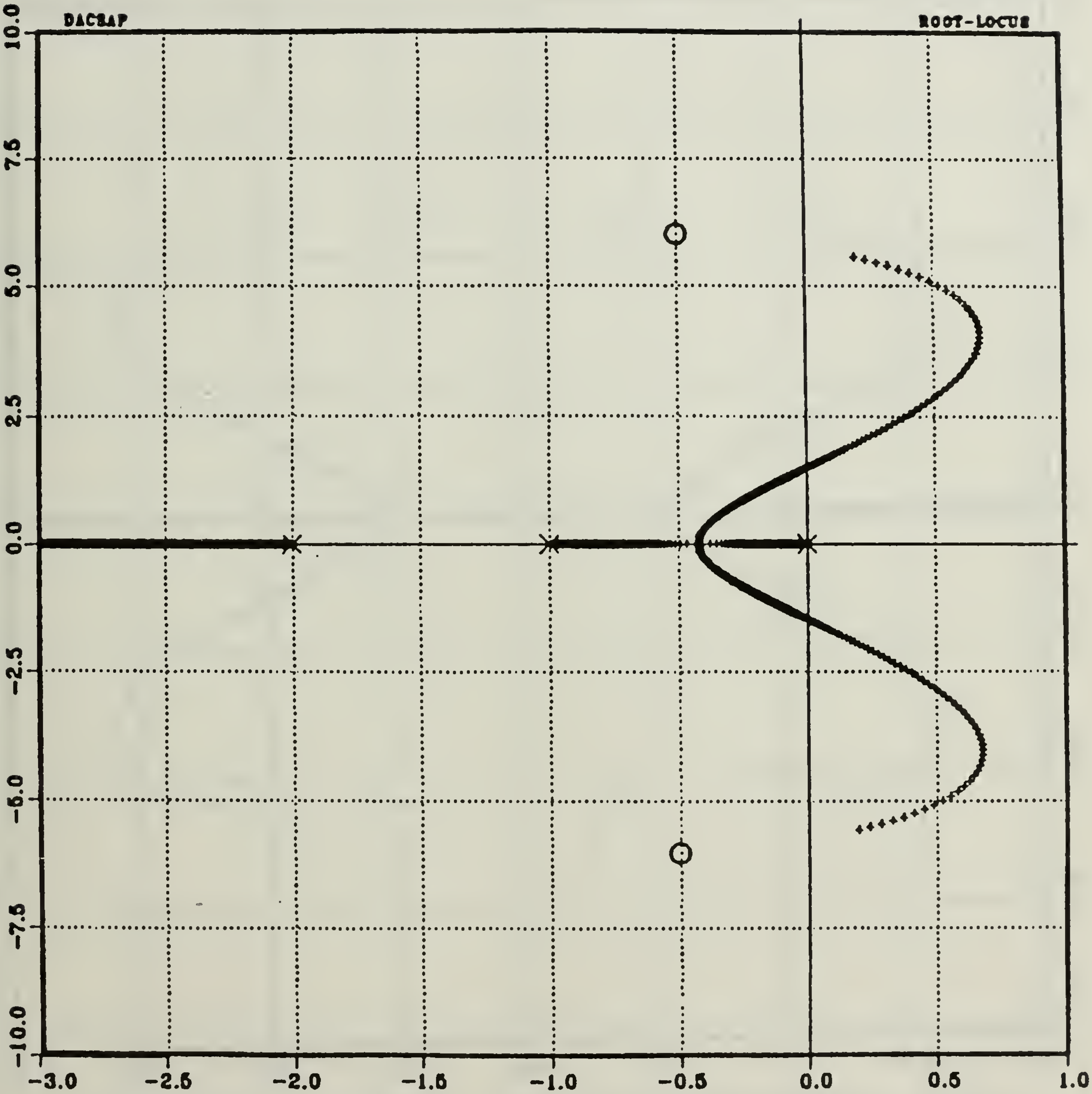

Figure 44. Compensated System Root Loci. Zeros at Location of Desired Roots ; -0.500 ∓ j6.000

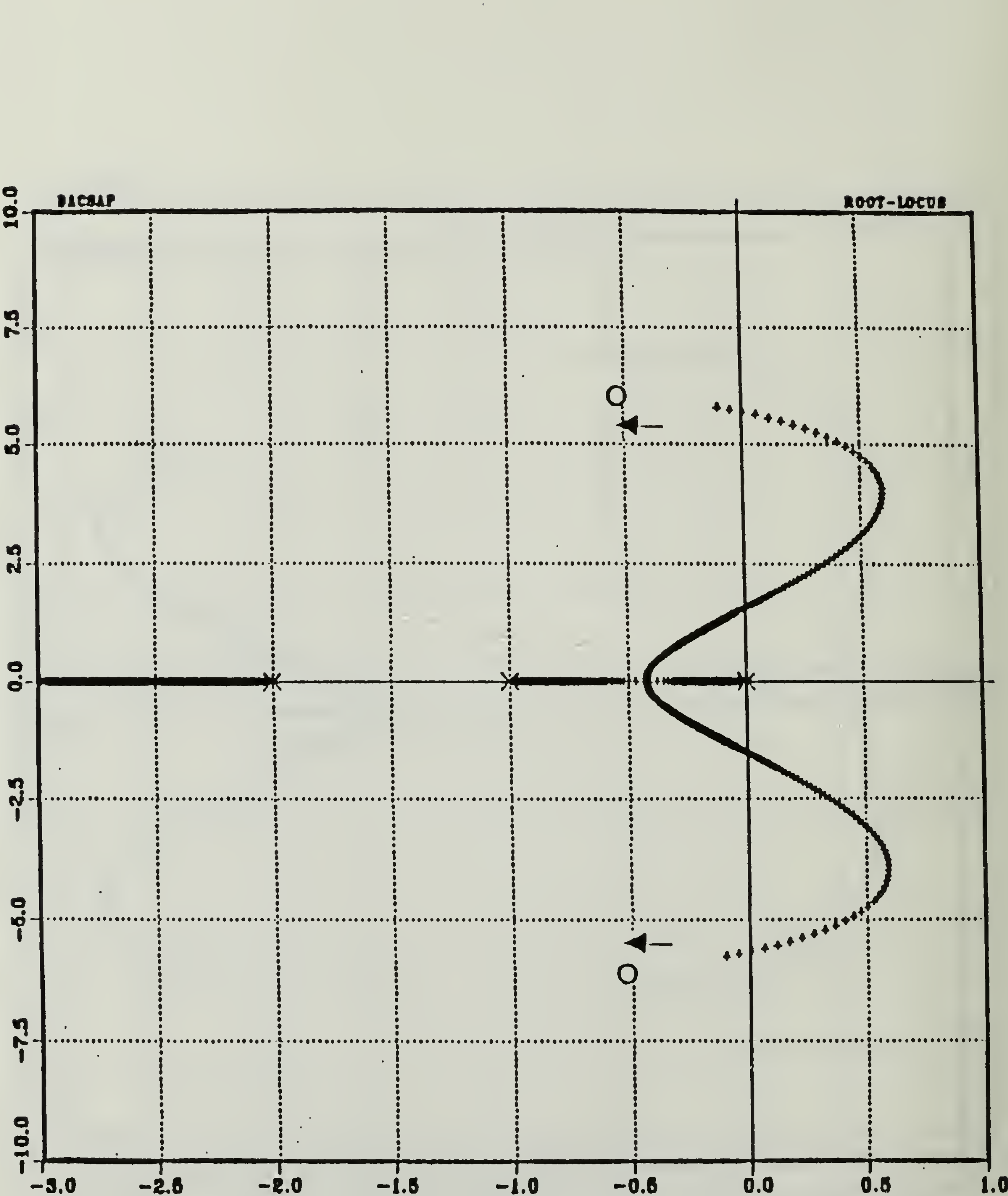

Figure 45. Offsetting Zeros by Matrix Methods. N-th Root at -100. Desired Roots; -0.500 ∓ j6.000

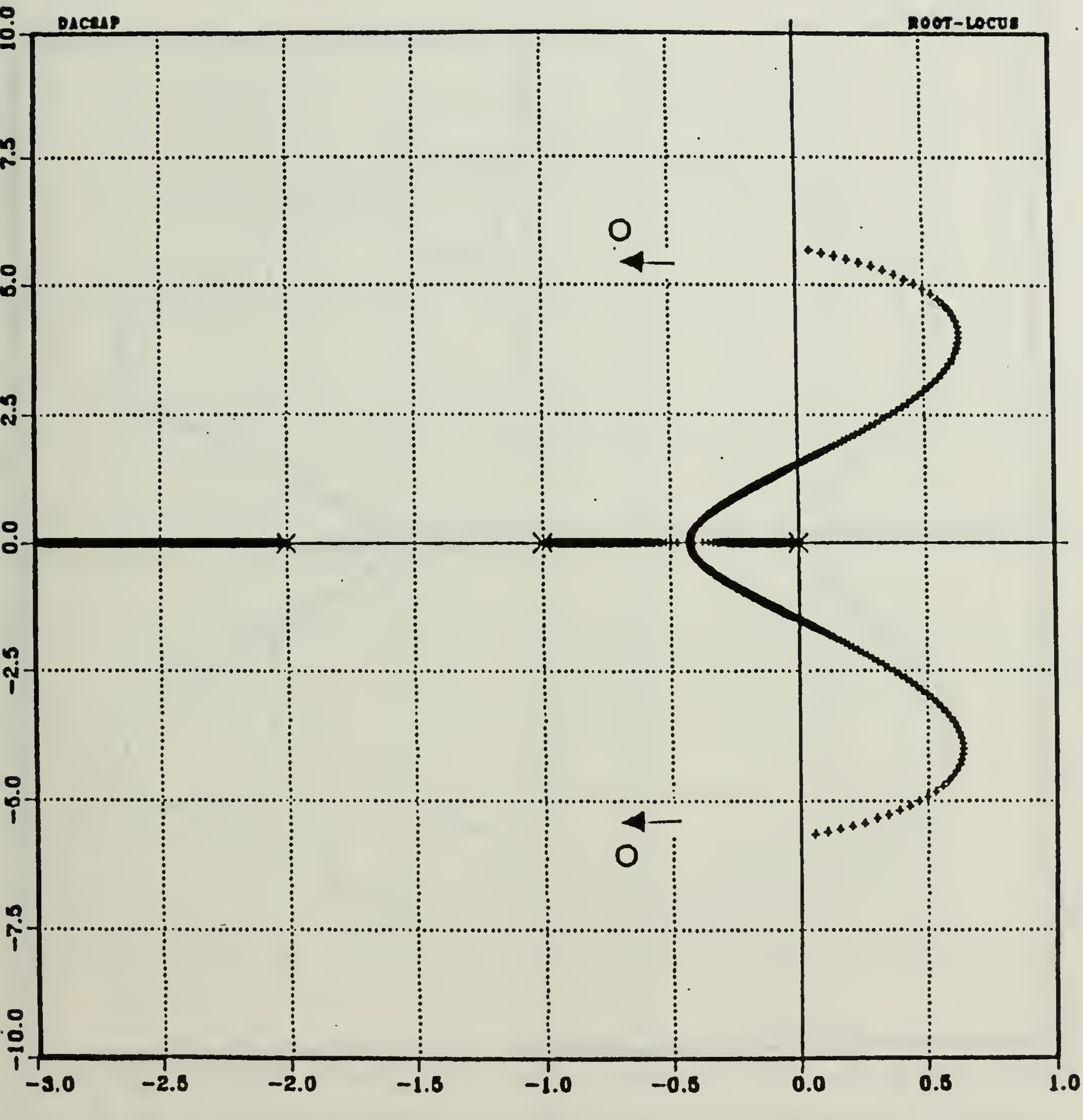

Figure 46. Offsetting Zeros by Matrix Methods. N-th Root at -50. Desired Roots; -0.500 ∓ j6.000

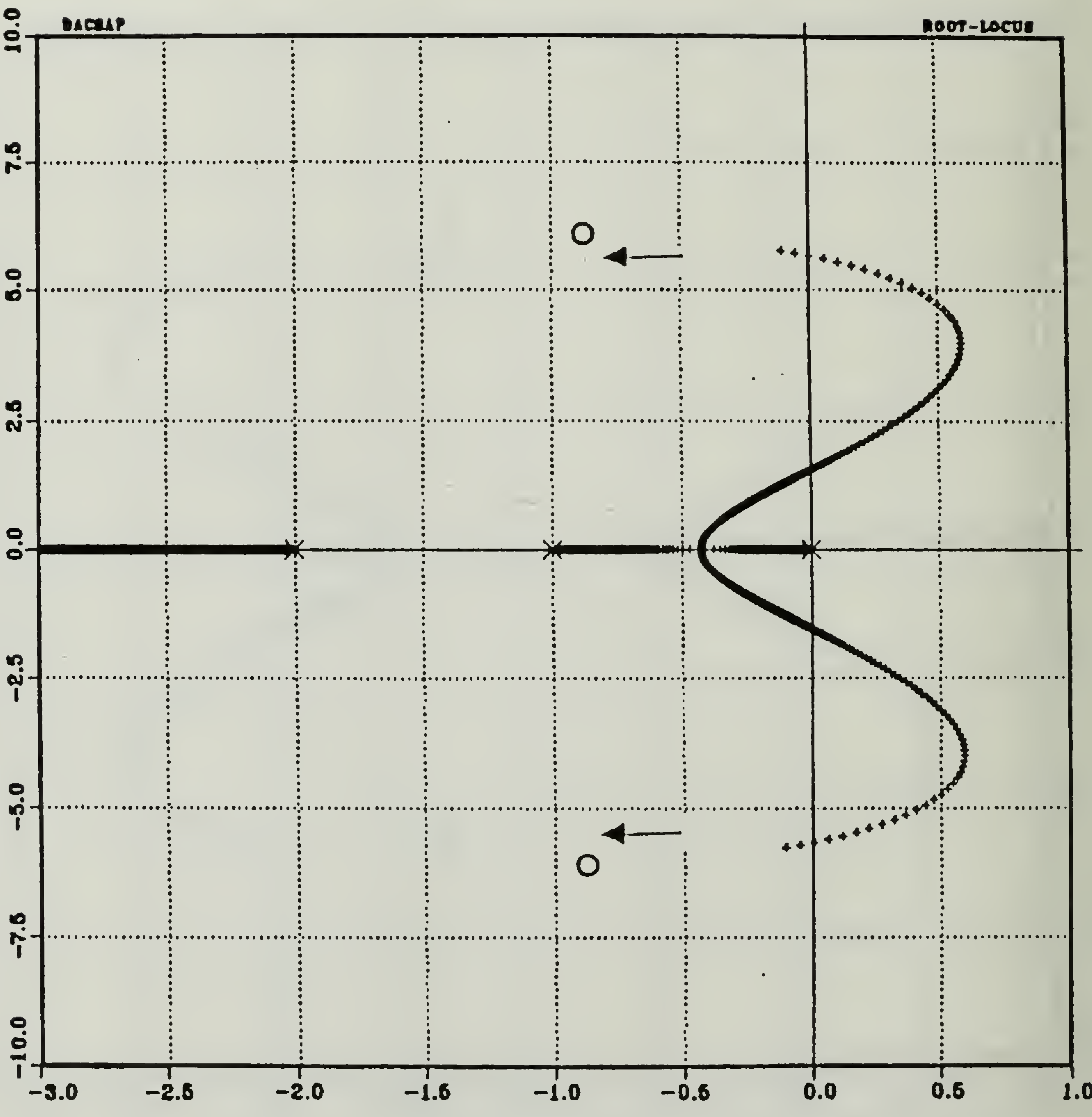

Figure 47. Offsetting Zeros by Matrix Methods. N-th Root at -30. Desired Roots; -0.500 ∓ j6.000

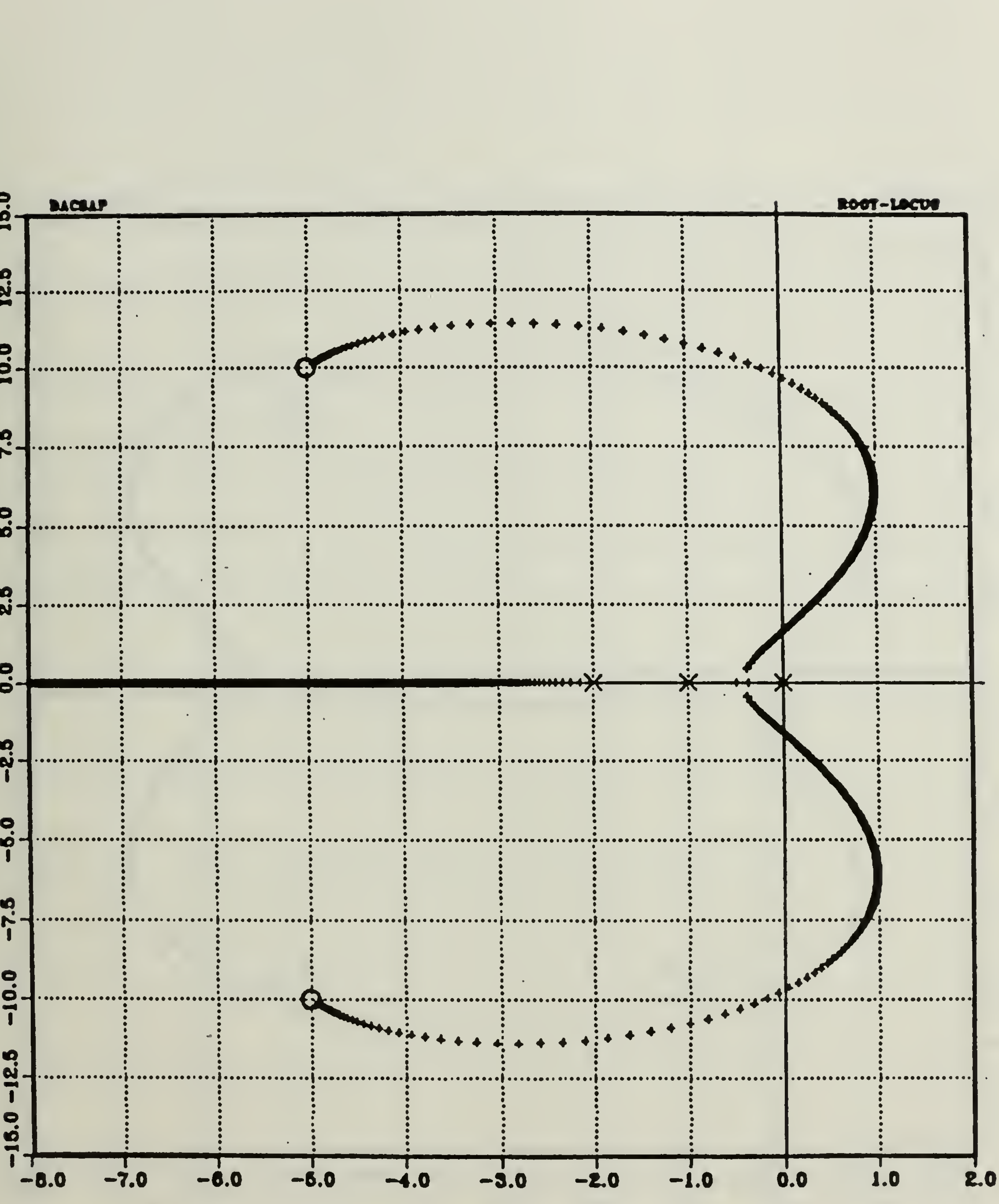

Figure 48. Compensated System Root Loci. Zeros at Location of Desired Roots; -5.000 ∓ j10.000

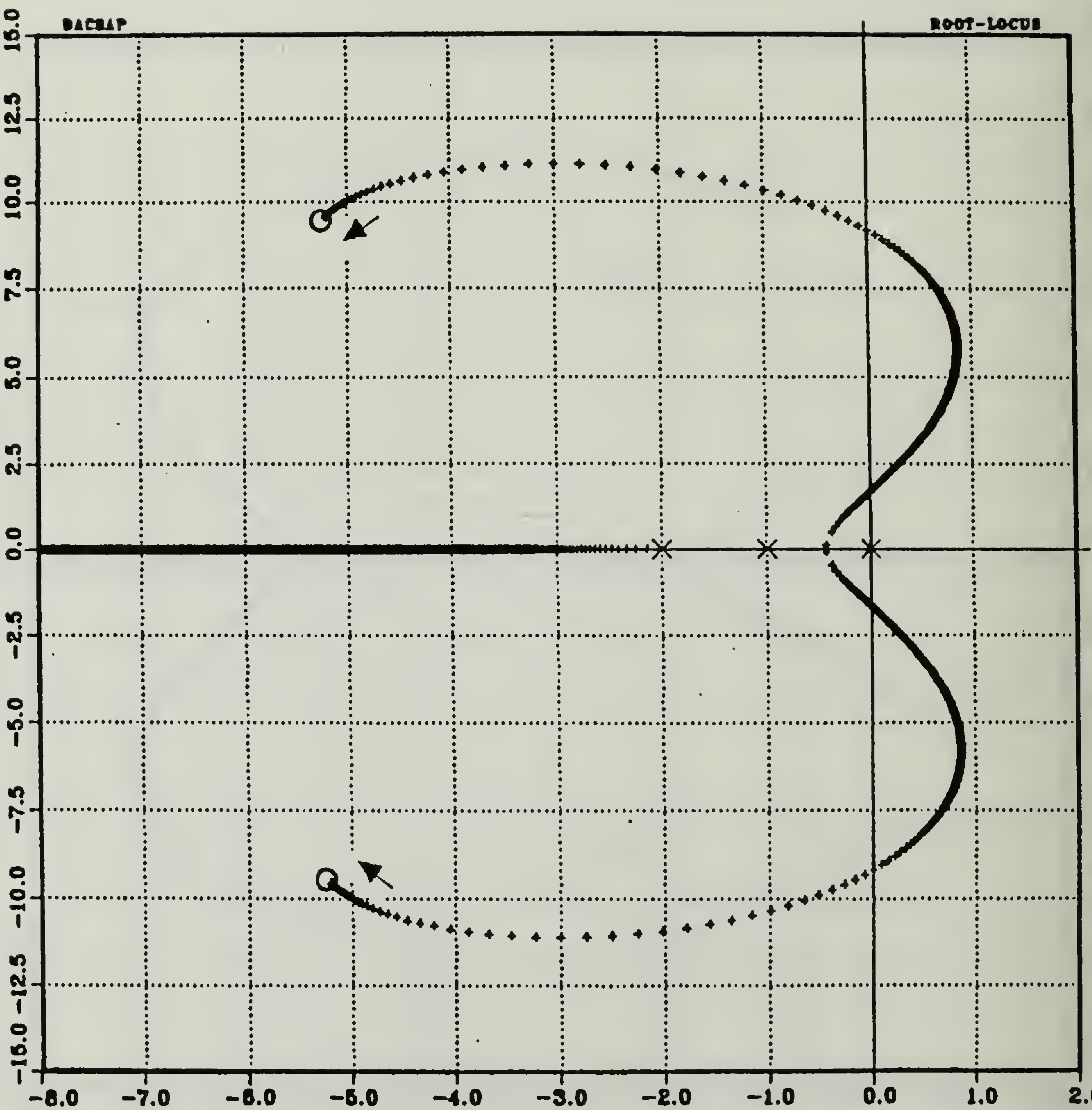

Figure 49. Offsetting Zeros by Matrix Methods. N-th Root at -100. Desired Roots; -5.000 ∓ j10.000

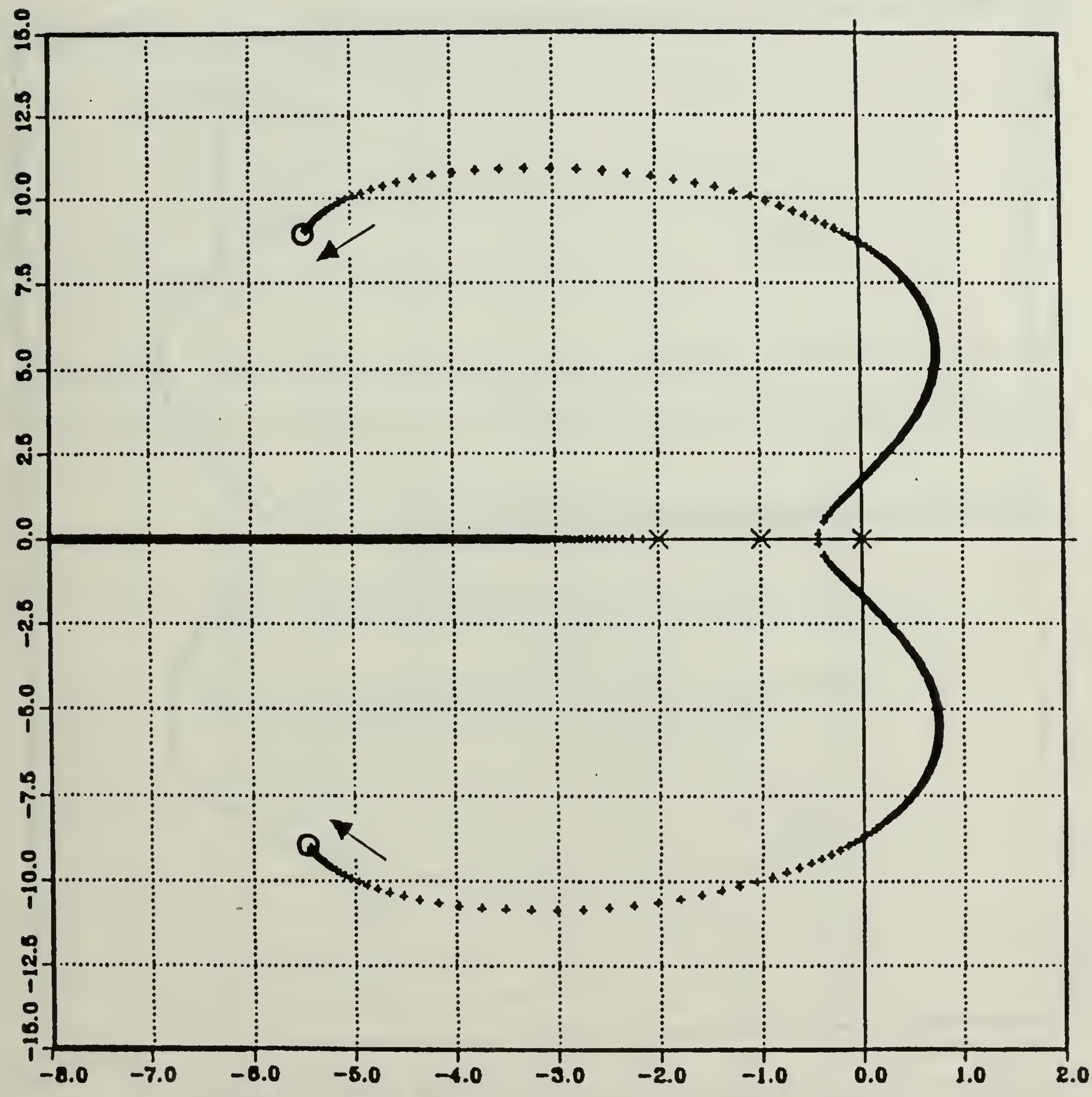

Figure 50. Offsetting Zeros by Matrix Methods. N-th Root at -50. Desired Roots; -5.000 ∓ j10.000

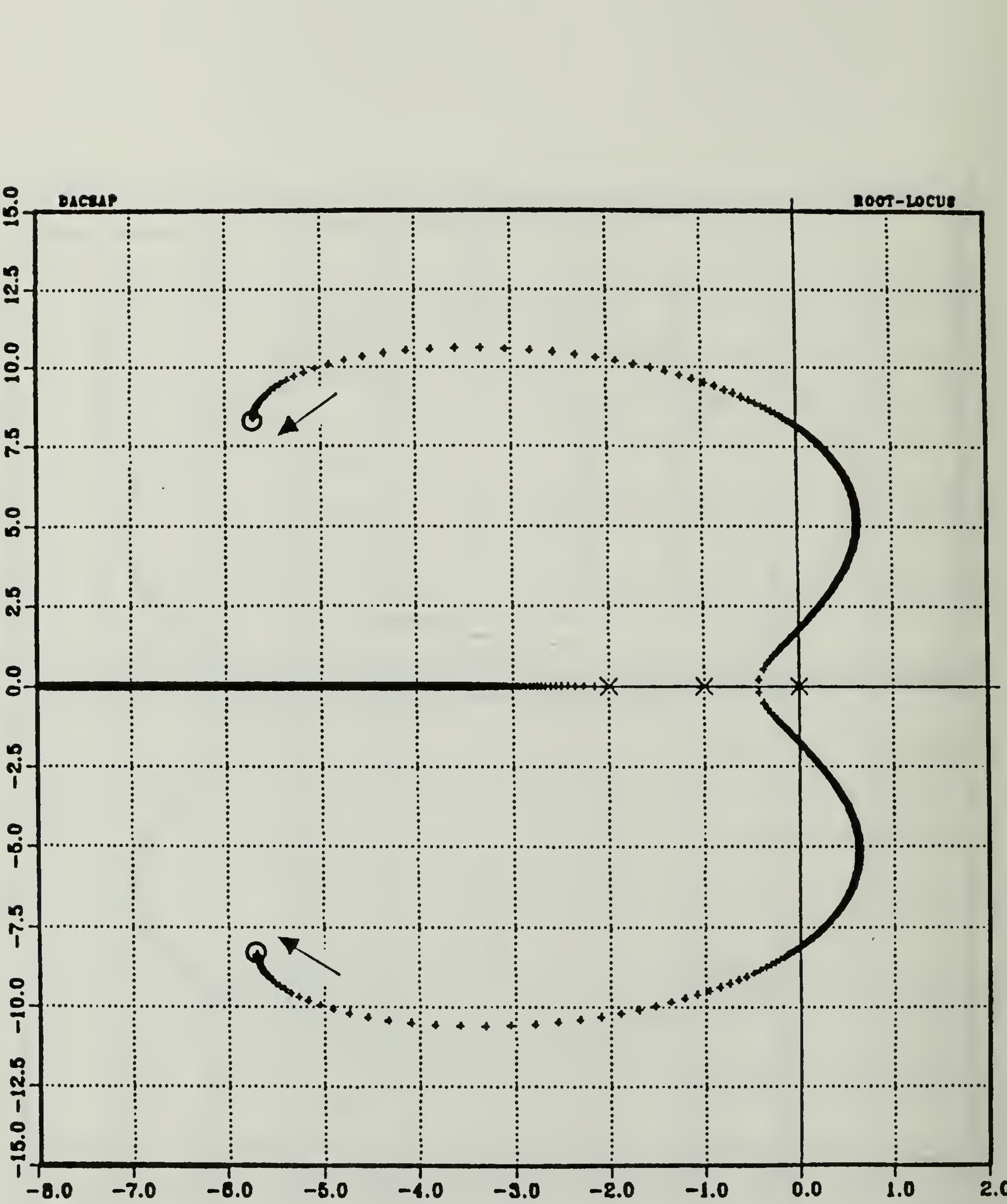

Figure 51. Offsetting Zeros by Matrix Methods. N-th Root at -30. Desired Roots; -5.000 ∓ j10.000

2. Plant 2

Transfer function:

$$G(s) = \frac{K}{(s+5)(s+3 \mp j5)} \tag{3.15}$$

The same procedure used for plant 1 is applied to plant 2.

The desired roots:

-1.000 ∓ j2.000

and

$K = 1000$

are specified. Table 7 shows the output of the SVS program, the feedback gains. The offset zero locations determined by the feedback gains are shown in Table 8 on page 64.

Table 7. THE FEEDBACK COEFFICIENTS AS A FUNCTION OF THE N-TH ROOT

$G(s) = \frac{K}{(s+5)(s+3 \pm j5)}$			
DESIRED ROOTS ; -1.000 ∓ j2.000			
LOCATION OF N-TH ROOTS	FEEDBACK COEFFICIENTS		
	k_0	k_1	k_2
-10000	49830	19941	9991
-1000	4830	1941	991
-100	330	141	91
-50	80	41	41

As seen from Table 7, the feedback coefficients decrease as the N-th root approaches the origin.

Table 8. OFFSET ZERO LOCATIONS BY MATRIX METHODS

$G(s) = \dfrac{K}{(s+5)(s+3 \pm j5)}$		
DESIRED ROOTS ; -1.000 ∓ j2.000		
LOCATION OF N-TH ROOTS	ZERO LOCATIONS	
	z_0	z_1
-10000	-0.99795	∓ j1.99789
-1000	-0.97932	∓ j1.97858
-100	-0.77472	∓ j1.73959
-50	-0.50000	∓ j1.30431

3. Plant 3

Transfer Function:

$$G(s) = \frac{K}{(s+1)(s+5)(s+10)(s+50)} \tag{3.16}$$

The same procedure used for plant 1 is applied to plant 3.

The desired roots:

-3.000 ∓ j5.000 , -25.000

and

$K = 1000$

are specified. Table 9 on page 65 shows the output of the SVS program, the feedback gains. The offset zero locations determined by the feedback gains are shown in Table 10 on page 65.

Table 9. THE FEEDBACK COEFFICIENTS AS A FUNCTION OF THE N-TH ROOT

$G(s) = \dfrac{K}{(s+1)(s+5)(s+10)(s+50)}$				
DESIRED ROOTS ; -3.000 ∓ j5.000 , -25.000				
LOCATION OF N-TH ROOTS	FEEDBACK COEFFICIENTS			
	k_0	k_1	k_2	k_3
-10000	6797500	1537380	259289	9960
-1000	677500	151380	25289	960
-100	65500	12780	1889	60
-50	31500	5080	589	10

Table 10. OFFSET ZERO LOCATIONS BY MATRIX METHODS

$G(s) = \dfrac{K}{(s+1)(s+5)(s+10)(s+50)}$			
DESIRED ROOTS ; -3.000 ∓ j5.000 , -25.000			
LOCATION OF N-TH ROOTS	ZERO LOCATIONS		
	z_0	z_1	z_2
-10000	-20.02734	-3.00284	∓ j5.00603
-1000	-20.28398	-3.029362	∓ j5.06116
-100	-24.63614	-3.42259	∓ j5.70882
-50	-50.00000	-4.45000	∓ j6.57248

4. Plant 4

Transfer Function:

$$G(s) = \frac{K}{s(s+1)(s+5)(s+10)(s+50)} \tag{3.17}$$

The same procedure used for plant 1 is applied to plant 4.

The desired roots:

$-3.000 \mp j5.000$

$-10.000 \mp j15.000$

and

$K = 1000$

are specified. Table 11 shows the output of the SVS program, the feedback gains. The offset zero locations determined by the feedback gains are shown in Table 12.

Table 11. THE FEEDBACK COEFFICIENTS AS A FUNCTION OF THE N-TH ROOT

$G(s) = \dfrac{K}{s(s+1)(s+5)(s+10)(s+50)}$					
DESIRED ROOTS ; $-3.000 \mp j5.000$, $-10.000 \mp j15.000$					
LOCATION OF N-TH ROOTS	FEEDBACK COEFFICIENTS				
	k_0	k_1	k_2	k_3	k_4
-10000	110500000	26308550	4789330	259614	9990
-1000	11050000	2638550	478330	26614	960
-100	1105000	271550	47230	2214	60
-50	552500	140050	23280	914	10

Table 12. OFFSET ZERO LOCATIONS BY MATRIX METHODS

$G(s) = \dfrac{K}{s(s+1)(s+5)(s+10)(s+50)}$				
DESIRED ROOTS ; $-3.000 \mp j5.000$, $-10.000 \mp j15.000$				
LOCATION OF N-TH ROOTS	ZERO LOCATIONS			
	z_0	z_1	z_2	z_3
-10000	-3.00272	$\mp$ j4.99735	-9.99057	$\mp$ j15.02007
-1000	-2.99598	$\mp$ j5.05312	-10.86547	$\mp$ j14.67926
-100	-3.13101	$\mp$ j4.78886	-15.31899	$\mp$ j18.10809
-50	-3.24178	$\mp$ j4.59785	-34.91377	-50.00000

5. Plant 5

Transfer Function:

$$G(s) = \frac{K}{s^2(s+1)(s+5)(s+10)(s+50)} \tag{3.18}$$

The same procedure is applied to plant 5.

The desired roots:

$-3.000 \mp j5.000$

$-10.000 \mp j15.000$

-20.000

and

$$K = 1000$$

are specified. Table 13 shows the output of the SVS program, the feedback gains. The offset zero locations determined by the feedback gains are shown in Table 14 on page 68.

Table 13. THE FEEDBACK COEFFICIENTS AS A FUNCTION OF THE N-TH ROOT

$G(s) = \frac{K}{s^2(s+1)(s+5)(s+10)(s+50)}$						
DESIRED ROOTS ;$-3.000 \mp j5.000$, $-10.000 \mp j15.000$, -20.000						
LOCATION OF N-TH ROOTS	FEEDBACK COEFFICIENTS					
	k_0	k_1	k_2	k_3	k_4	k_5
-10000	22.1×10^8	63.7×10^7	12.2×10^7	9.9×10^6	4.6×10^5	9980
-1000	1.1×10^8	31.8×10^7	6.1×10^7	5.0×10^6	2.3×10^5	4980
-100	0.2×10^8	6.4×10^7	1.2×10^7	1.0×10^6	0.5×10^5	980
-50	0.1×10^8	0.3×10^7	0.1×10^7	0.1×10^6	0.2×10^4	30

Table 14. OFFSET ZERO LOCATIONS BY MATRIX METHODS

$G(s) = \dfrac{K}{s^2(s+1)(s+5)(s+10)(s+50)}$					
DESIRED ROOTS ; -3.000 ∓ j5.000 , -10.000 ∓ j15.000 , -20.000					
LOCATION OF N-TH ROOTS	ZERO LOCATIONS				
	z_0	z_1	z_2	z_3	z_4
-10000	-2.99992	∓ j4.99721	-10.03563	∓ j14.98211	-20.03377
-1000	-2.99136	∓ j4.99713	-10.35609	∓ j14.80188	-20.35609
-100	-2.92064	∓ j4.96477	-13.00504	∓ j11.64400	-27.32364
-50	-2.85063	∓ j4.91767	-12.71603	∓ j8.14466	-50.00000

6. Plant 6

Transfer Function:

$$G(s) = \frac{K}{s(s+1)(s+5)^2(s+10)(s+50)} \tag{3.19}$$

The same procedure is applied to plant 6.

The desired roots:

-3.000 ∓ j5.000

-10.000 ∓ j15.000

-20.000

and

$K = 1000$

are specified. Table 15 on page 69 shows the output of the SVS program, the feedback gains. The offset zero locations determined by the feedback gains are shown in Table 16 on page 69.

Table 15. THE FEEDBACK COEFFICIENTS AS A FUNCTION OF THE N-TH ROOT

$G(s) = \dfrac{K}{s(s+1)(s+5)^2(s+10)(s+50)}$						
DESIRED ROOTS ; -3.000 ∓ j5.000 , -10.000 ∓ j15.000 , -20.000						
LOCATION OF N-TH ROOTS	FEEDBACK COEFFICIENTS					
	k_0	k_1	k_2	k_3	k_4	k_5
-10000	22.1×10^8	63.7×10^7	12.2×10^7	10.0×10^5	46.0×10^4	9975
-1000	2.2×10^8	6.4×10^7	1.2×10^7	1.0×10^5	4.6×10^4	975
-100	0.2×10^8	0.7×10^7	0.2×10^7	0.1×10^6	0.4×10^5	75
-50	0.1×10^8	0.3×10^7	0.1×10^7	0.1×10^6	0.2×10^4	25

Table 16. OFFSET ZERO LOCATIONS BY MATRIX METHODS

$G(s) = \dfrac{K}{s(s+1)(s+5)^2(s+10)(s+50)}$					
DESIRED ROOTS ; -3.000 ∓ j5.000 , -10.000 ∓ j15.000 , -20.000					
LOCATION OF N-TH ROOTS	ZERO LOCATIONS				
	z_0	z_1	z_2	z_3	z_4
-10000	-2.99828	∓ j5.00183	-10.06383	∓ j14.99751	-19.97114
-1000	-2.98252	∓ j5.01791	-10.65976	∓ j14.97131	-19.69388
-100	-2.79874	∓ j5.14077	-19.30504	∓ j14.83314	-14.51175
-50	-2.58322	∓ j5.19193	-34.56360	∓ j11.72600	-9.86628

C. CONCLUSIONS

As seen from the tables, the SVS program chooses the offset zero locations near to the desired root locations if the N-th root is far out on the negative real axis, but the offset may be large if the N-th root is not large. The common precedure is to extend the path of the root loci past the desired dominant locations. Only the zeros that determine the dominant roots are offset by a large amount. The gains are decreased as the N-th root moves close to the origin, which may be important.

IV. THE MOVEMENT OF THE ZEROS TO OFFSET POINTS AND GUIDELINES FOR DESIGN

A. THE MOVEMENT OF THE ZEROS TO OFFSET POINTS

In using the transfer function method, if the zeros of H(s) are placed exactly at the desired root locations, then the loop gain required to drive the roots to these zeros is very high. By placing the zeros at locations slightly offset from the the desired root locations, we should be able to drive the roots to the desired locations with a much lower loop gain. The offset zero locations are on (or near) an extension of the root locus through the desired root point. It is not known how far to move the zeros. Therefore we choose arbitary points which are shown in Figure 52 on page 71 to study the problem. By using the computer, for each offset zeros case, we obtain:

1. The adjustable gain, Kk_2 , at or near the desired points

2. The locations of roots near the desired points

3. The feedback gains (k_0, k_1, k_2, k_3, ..., k_{n-1})

4. The DC gains

5. The error coefficient gains

6. The settling time

7. The % overshoot

which are calculated and shown in Table 17 on page 72. As seen from Table 17 on page 72, the gain that we need to put the roots at desired locations is decreased when the zeros are offset. We do not need to move the zeros too far away. Therefore the study was repeated with chosen zero locations much closer to the desired root points. The new set of chosen zeros is indicated in Figure 53. The result is given in Table 18 on page 73.

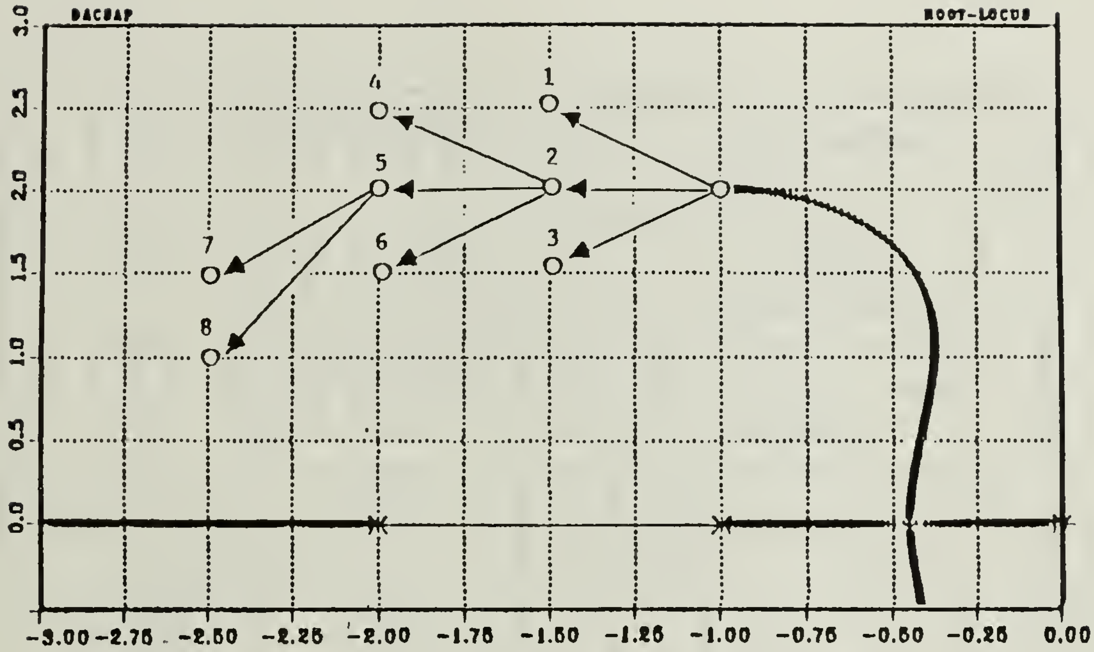

Figure 52. Arbitrarily Chosen Offset Zero Location (I)

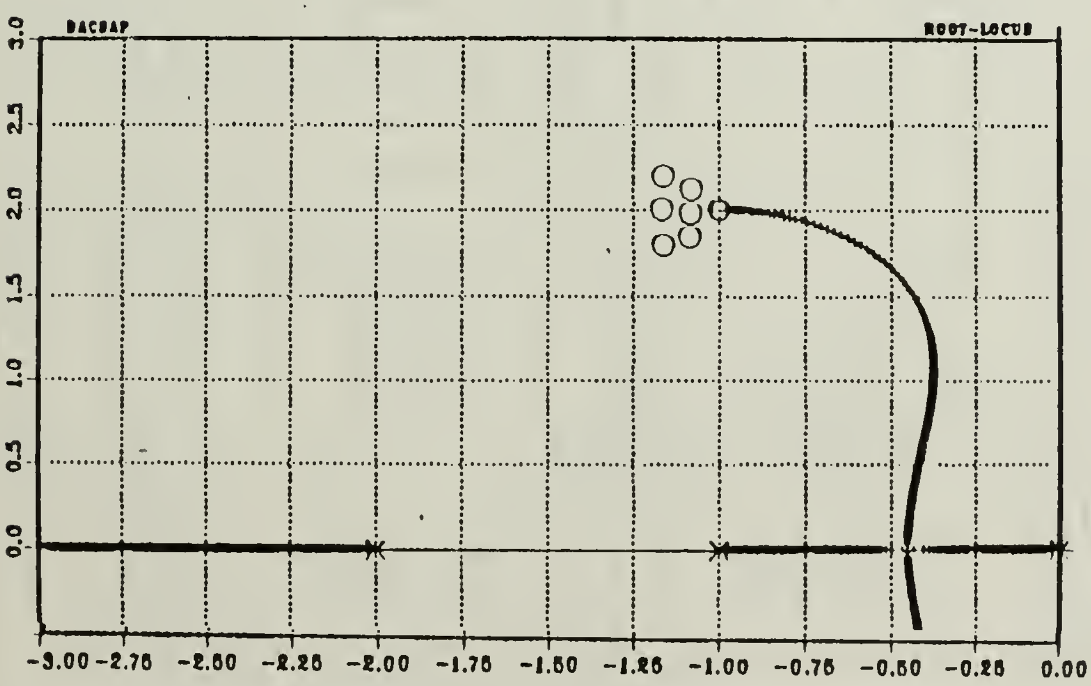

Figure 53. Arbitrarily Chosen Offset Zero Locations (II)

Table 17. ARBITRARILY CHOSEN OFFSET ZERO LOCATIONS (I)

$G(s) = \frac{K}{s(s+1)(s+2)}$ $H(s) = k_2 s^2 + k_1 s + k_0$ DES. ROOTS; -1.000 ∓ j2.000

NUMBER	OFFSET ZEROS LOCATIONS	ROOT LOCUS GAIN, (Kk_2)	ROOTS AT CLOSEST POINT TO DESIRED LOCATION	$k_0(\times 10^{-4})$	$k_1(\times 10^{-4})$	$k_2(\times 10^{-4})$	$k_3(\times 10^{-4})$	DC GAIN	ERROR COEF. GAIN	SETTLING TIME(sec.)	% OVERSHOOT
1	-1.5 ±j 2.5	7.541	-8.535 -1.000 ±j 2.551	64.09	22.62	7.54	-	32.04	2.603	~4	15
2	-1.5 ±j 2.0	5.201	-6.208 -0.996 ±j 2.051	32.50	15.60	5.20	-	22.10	2.511	~4	14
3	-1.5 ±j 1.5	3.552	-4.538 -1.005 ±j 1.584	15.98	10.65	3.55	-	7.99	1.263	~4	11
4	-2.0 ±j 2.5	4.141	-5.185 -1.002 ±j 2.691	42.44	16.56	4.14	-	21.22	2.287	~4	24
5	-2.0 ±j 2.0	3.030	-4.017 -1.006 ±j 2.241	24.24	12.12	3.03	-	12.12	1.717	~4	22
6	-2.0 ±j 1.5	2.121	-3.127 -0.998 ±j 1.875	13.25	8.48	2.12	-	6.63	1.264	~4	17
7	-2.5 ±j 1.5	1.801	-2.823 -0.988 ±j 2.107	15.30	9.00	1.80	-	7.65	1.391	~4	23
8	-2.5 ±j 1.0	1.401	-2.413 -1.993 ±j 1.794	10.15	7.00	1.40	-	5.08	1.127	~4	20

Table 18. ARBITARY CHOSEN OFFSET ZERO LOCATIONS (II)

$G(s) = \frac{K}{s(s+1)(s+2)}$ $H(s) = k_2 s^2 + k_1 s + k_0$ DES. ROOTS; -1.000 ∓ j2.000

NUMBER	OFFSET ZEROS LOCATIONS	ROOT LOCUS GAIN, (Kk_2)	ROOTS AT CLOSEST POINT TO DESIRED LOCATION	$k_0(\times 10^{-4})$	$k_1(\times 10^{-4})$	$k_2(\times 10^{-4})$	$k_3(\times 10^{-4})$	DC GAIN	ERROR COEF. GAIN	SETTLING TIME(sec.)	% OVERSHOOT
1	-1.1 ± j 2.1	27.043	-28.043 -0.9997 ± j 2.1021	151.90	59.48	27.04	-	75.95	2.471	~ 4	5
2	-1.1 ± j 2.0	24.995	-25.998 -0.9998 ± j 2.0024	130.19	54.98	24.99	-	65.09	2.285	~ 4	5
3	-1.1 ± j 2.1	23.042	-24.042 -0.9998 ± j 1.9019	111.06	50.69	23.04	-	55.53	2.108	~ 4	5
4	-1.1 ± j 2.2	14.585	-15.699 -0.9430 ± j 1.8944	70.30	32.09	14.59	-	35.15	2.062	~ 4	8
5	-1.1 ± j 2.0	12.484	-13.488 -0.9981 ± j 2.0090	67.91	29.96	12.49	-	33.95	2.125	~ 4	8
6	-1.2 ± j 1.9	11.513	-12.513 -0.9979 ± j 1.9102	58.12	27.62	11.51	-	29.06	1.838	~ 4	8

The results of Table 17 on page 73 and Table 18 on page 74 are:

1. A little movement of the zero location will decrease the gain needed to put the roots at desired locations.

2. The system DC gain and Error Coefficient Gains change for each zero location and decrease as the zeros are moved away from the desired point.

B. GUIDELINES FOR DESIGN

Root movement as a function of gain is observed in Chapter II and offsetting the zeros by matrix methods is studied in Chapter III. From these we can obtain some guidelines for design. These guidelines are:

1. In order to find in which direction the zeros must be moved, draw the root locus curve of the compensated plant, and extend it from the zeros in the natural direction. If the new root locus does not pass through the desired point, choose a new offset point by moving the zeros a little from the offset location until the root locus does pass through the desired point or at an acceptable distance from the desired point.

2. When the zeros are relocated, it is not necessary to move all the zeros, but only the dominant zeros.

3. The real part of the N-th root should not be moved closer than a distance which is ten times larger than the real part of the desired roots.

4. To find how far the zeros must be moved, estimate the length of the root locus curve of the compensated plant from the pole to the zero, and move the zero about 2% to 3% of the curve length. This will put the roots at desired locations by decreasing the loop gain to about 10% of the original value. When the offset location is chosen far away, the gain at the desired location will be decreased. How much decrease is needed depends on the designer.

V. APPLICATIONS OF THE GUIDELINES TO ALL-POLE PLANTS

In this chapter the design guidelines obtained from the preceding studies are applied to high order all-pole plants to determine their effectiveness.

A. PLANT 1

The plant transfer function is:

$$G(s) = \frac{K}{s(s+1)(s+5)(s+10)(s+50)} \tag{5.1}$$

For this plant, two different sets of desired roots are studied.

The desired roots are:

a. -3.000 ∓ j5.000 , -15.000 ∓ j5.000
b. -3.000 ∓ j5.000 , -15.000 , -30.000

For each set of desired roots, the guidelines obtained previously will be applied.

Case A

The desired roots are:

-3.000 ∓ j5.000 , -15.000 ∓ j5.000

The closed loop transfer function is:

$$G_{eq}(s) = \frac{G(s)}{1 + G(s)H(s)} \tag{5.2}$$

where,

$$G(s)H(s) = \frac{K(s+3\mp j5)(s+15\mp j5)}{s(s+1)(s+5)(s+10)(s+50)} \tag{5.3}$$

The characteristic equation is:

$$1 + G(s)H(s) = 0 \tag{5.4}$$

$$G(s)H(s) = \frac{K(s+3\mp j5)(s+15\mp j5)}{s(s+1)(s+5)(s+10)(s+50)} \tag{5.5}$$

The forward gain, K, is chosen as 1000, and the numerator of the expression above can be written as:

$$NUMERATOR = Kk_4(s^4 + \frac{k_3}{k_4} s^3 + \frac{k_2}{k_4} s^2 + \frac{k_1}{k_4} s + \frac{k_0}{k_4}) \quad (5.6)$$

Kk_4 determines the loop gain of the original compensated system. Since k_4 equals 1.0, the initial loop gain is equal to 1000, and the locations of all roots are given below.

-2.96870 ∓ j5.14918
-14.69699 ∓ j5.14923
-1030.66943

These are acceptable. Because a small gain is needed, the guidelines are applied twice. In other words, the root locus curve of the compensated plant is extended about 2% to 3 % in the its natural direction, and only dominant zeros are relocated.

The result of the first application is: (Figure 55 on page 86)

OFFSET ZERO LOC.	ROOTS	LOOP GAIN
-3.25 ∓ j4.75	-2.94454 ∓ j5.35373 -12.95768 -135.32245	106.36934

As seen from the results, the extended root locus curve is at an acceptable distance from the desired roots, so that the error of the location of the roots is about 2%. The initial loop gain is decreased to

$$\frac{106.36934}{1000} \times 100 = 10.6\%. \quad (5.7)$$

The N-th root is located at a distance which is nine times larger than the real part of the desired roots.

The result of a second application (i.e., another extension) is: (Figure 56 on page 87)

OFFSET ZERO LOC.	ROOTS	LOOP GAIN
-3.50 ∓ j4.50	-2.97463 ∓ j4.85538 -11.98653 -94.12236	13.23991

As seen from the results, the extended root locus curve is at an acceptable distance from the desired roots, so that the error of the location of the roots is about 1%. The loop gain is decreased to

$$\frac{13.23991}{106.36934} \times 100 = 12.4\% \tag{5.8}$$

of the gain determined by the first offset. The final loop gain is

$$\frac{13.23991}{1000} \times 100 = 1.32\% \tag{5.9}$$

of the initial loop gain. The N-th root is located at a distance which is six times larger than the real part of the desired roots.

Case B

The desired roots are:

-3.000 ∓ j5.000 , -15.000 , -30.000

The closed loop transfer function is:

$$G_{eq}(s) = \frac{G(s)}{1 + G(s)H(s)} \tag{5.10}$$

where,

$$G(s)H(s) = \frac{K(s + 3 \mp j5)(s + 15)(s + 15)}{s(s + 1)(s + 5)(s + 10)(s + 50)} \tag{5.11}$$

The characteristic equation is:

$$1 + G(s)H(s) = 0 \tag{5.12}$$

$$G(s)H(s) = \frac{K(s+3\mp j5)(s+15)(s+15)}{s(s+1)(s+5)(s+10)(s+50)} \tag{5.13}$$

The compensated system root loci are given in Figure 54 on page 88. The forward gain, K, is chosen as 1000 again, and Kk_4 determines the loop gain of the original compensated system. Since k_4 equals 1.0, the initial loop gain is equal to 1000, and the locations of all roots are given below:

$$\begin{array}{r} -2.98497 \mp j5.01218 \\ -15.15249 \\ -29.21259 \\ -1015.66943 \end{array}$$

These are acceptable. Since a small gain is needed, the guidelines are applied again. In other words, the root locus curve of the compensated plant is extended about 2% to 3 % in its natural direction, and only dominant zeros are relocated.

The result of this application is: (Figure 57 on page 89)

OFFSET ZERO LOC.	ROOTS	LOOP GAIN
-3.15 ∓ j4.85	-2.98646 ∓ j4.98617 -18.64367 ∓ j2.95003 -135.32245	92.36934

For this application, the extended root locus curve is at an acceptable distance from the desired roots, so that the error of the location of the roots is about 1%. The initial loop gain is decreased to

$$\frac{92.36934}{1000} \times 100 = 9.24\%. \tag{5.14}$$

The N-th root is located at a distance which is less than ten times the real part of the dominant roots, but, is probably acceptable.

For this plant, the guidelines are working.

B. PLANT 2

The rules obtained will be applied to the 4-th order plant.

The plant transfer function is:

$$G(s) = \frac{K}{(s+2)(s+20)(s^2+2s+100)} \tag{5.15}$$

The desired roots are:

-10.000 ∓ j5.000 , -25.000

The closed loop transfer function is:

$$G_{eq}(s) = \frac{G(s)}{1 + G(s)H(s)} \tag{5.16}$$

where,

$$G(s)H(s) = \frac{K(s+10\mp j5)(s+25)}{(s+2)(s+20)(s^2+2s+100)} \tag{5.17}$$

The characteristic equations is:

$$1 + G(s)H(s) = 0 \tag{5.18}$$

$$G(s)H(s) = \frac{K(s+10\mp j5)(s+25)}{(s+2)(s+20)(s^2+2s+100)} \tag{5.19}$$

The compensated system root loci are given in Figure 59 on page 90. The forward gain, K, is chosen as 1000, and the numerator of the expression above can be written as:

$$NUMERATOR = Kk_3(s^3 + \frac{k_2}{k_3}s^2 + \frac{k_1}{k_3}s + \frac{k_0}{k_3}) \tag{5.20}$$

Kk_3 determines the loop gain of the original compensated system. Since k_3 equals 1.0, the initial loop gain is equal to 1000, and the locations of all roots are given below:

-25.00701

$-10.00069 \mp j4.99993$

-9999.68743

These are acceptable. Since a small gain is needed, the guidelines are applied three times. In other words, the root loci of the compensated plant is extended three times about 2% to 3% each time in the its natural direction and only dominant zeros are re-located.

The result of the first application is: (Figure 60 on page 91)

OFFSET ZERO LOC.	ROOTS	LOOP GAIN
$-9.90 \mp j5.10$	$-10.04356 \mp j5.00452$ -25.05678 -8288.22450	98.38924

As seen from the results, the extended root loci are at an acceptable distance from the desired roots, so that the error of the location of the roots is about 1%. The initial loop gain is decreased to

$$\frac{98.38924}{1000} \times 100 = 9.84\%. \qquad (5.21)$$

The result of a second application (i.e., another extension): (Figure 61 on page 92)

OFFSET ZERO LOC.	ROOTS	LOOP GAIN
$-9.50 \mp j5.50$	$-10.12455 \mp j5.09537$ -27.05678 -1256.22450	15.36934

As seen from the results, the extended root locus curve is at an acceptable distance from the desired roots, so that the error of the location of the roots is about 2%. The loop gain is decreased to

$$\frac{15.36934}{98.38924} \times 100 = 15.6\% \qquad (5.22)$$

of the gain determined by the first offset. The final loop gain is

$$\frac{15.36934}{1000} \times 100 = 1.54\% \tag{5.23}$$

of the initial loop gain.

The result of a third application (i.e., another extension) is: (Figure 62 on page 93)

OFFSET ZERO LOC.	ROOTS	LOOP GAIN
-9.00 ∓ j5.00	-10.34005 ∓ j5.31537 -39.05678 ∓ j10.88389	0.78693

As seen from the results, the extended root locus curve is at an acceptable distance from the desired roots, so that the error of the location of the roots is about 3.4%. The loop gain is decreased to

$$\frac{0.78693}{15.36934} \times 100 = 5.6\% \tag{5.24}$$

of the gain determined by the second offset. The final loop gain is

$$\frac{0.78693}{1000} \times 100 = 0.08\% \tag{5.25}$$

of the initial loop gain.

C. PLANT 3

The rules obtained will be applied to the 7-th order plant.

The plant transfer function is:

$$G(s) = \frac{K}{s^2(s+10)(s+100)(s+500)(s^2+50s+250)} \tag{5.26}$$

The desired roots are:

-5.000 ∓ j10.000

-50.000 ∓ j15.000

$-300.000 \mp j200.000$

The closed loop transfer function is:

$$G_{eq}(s) = \frac{G(s)}{1 + G(s)H(s)} \tag{5.27}$$

where,

$$G(s)H(s) = \frac{K(s + 5 \mp j10)(s + 50 \mp j15)(s + 300 \mp j200)}{s^2(s + 10)(s + 100)(s + 500)(s^2 + 50s + 250)} \tag{5.28}$$

The characteristic equation is:

$$1 + G(s)H(s) = 0 \tag{5.29}$$

$$G(s)H(s) = \frac{K(s + 5 \mp j10)(s + 50 \mp j15)(s + 300 \mp j200)}{s^2(s + 10)(s + 100)(s + 500)(s^2 + 50s + 250)} \tag{5.30}$$

The compensated system root loci are given in Figure 63 on page 94. The forward gain, K, is chosen as 1000, and the numerator of the expression above can be written as:

$$NUMERATOR = Kk_6(s^6 + \frac{k_5}{k_6}s^5 + \frac{k_5}{k_6}s^4 + \frac{k_3}{k_6}s^3 + \frac{k_2}{k_6}s^2 + \frac{k_1}{k_6}s + \frac{k_0}{k_6}) \tag{5.31}$$

Kk_6 determines the loop gain of the original compensated system. Since k_6 equals 1.0, the initial loop gain is equal to 1000. The locations of all roots are given below:

$-5.00003 \mp j9.99993$

$-49.99993 \mp j15.00993$

-355.45343 ∓ 223.58589

These are acceptable. Since a small gain is needed, the guidelines are applied three times. In other words, the root locus curve of the compensated plant is extended three times, about 2% to 3 % each time, in the natural direction, and only dominant zeros are relocated.

The result of the first application is: (Figure 64 on page 95)

OFFSET ZERO LOC.	ROOTS	LOOP GAIN
-5.01 ∓ j10.01	-4.99834 ∓ j10.00004 -50.00334 ∓ j14.99894 -299.57056 ∓ j200.34682 -90840.75105	90.89308

As seen from the results, the extended root locus curve is at an acceptable distance from the desired roots, so that the error of the location of the roots is about 1%. The initial loop gain is decreased to

$$\frac{90.89308}{1000} \times 100 = 9.09\%. \tag{5.32}$$

The result of a second application (i.e., another extension) is: (Figure 65 on page 96)

OFFSET ZERO LOC.	ROOTS	LOOP GAIN
-5.01 ∓ j10.00	-4.99653 ∓ j10.00006 -50.00346 ∓ j14.96854 -295.57158 ∓ j208.34573 -8760.87038	8.78507

As seen from the results, the extended root locus curve is at an acceptable distance from the desired roots, so that the error of the location of the roots is about 1%. The loop gain is decreased to

$$\frac{8.78507}{90.89308} \times 100 = 9.67\% \tag{5.33}$$

of the gain determined by the first offset. The final loop gain is

$$\frac{8.78507}{1000} \times 100 = 0.88\% \tag{5.34}$$

of the initial loop gain.

The result of a third application (i.e., more extension) is: (Figure 66 on page 97)

OFFSET ZERO LOC.	ROOTS	LOOP GAIN
$-5.04 \mp j10.02$	$-5.00046 \mp j10.00123$ $-50.01675 \mp j14.70563$ $-295.21086 \mp j208.14592$ -543.68905	0.87045

As seen from the results, the extended root locus curve is at an acceptable distance from the desired roots, so that the error of the location of the roots is about 1%. The loop gain is decreased to

$$\frac{0.87045}{8.78507} \times 100 = 9.91\% \tag{5.35}$$

of the gain determined by the second offset. The final loop gain is

$$\frac{0.87045}{1000} \times 100 = 0.08\% \tag{5.36}$$

of the initial loop gain.

D. CONCLUSIONS

In general, the guidelines to estimate the offset zero locations are working. Only dominant zeros must be relocated in the direction of the root loci. The guideline that estimates the decreases of the gain may not work for some plants. If that happens zeros that attract the system dominant roots are offset to maintain the system error coefficient by the designer using trial and error methods.

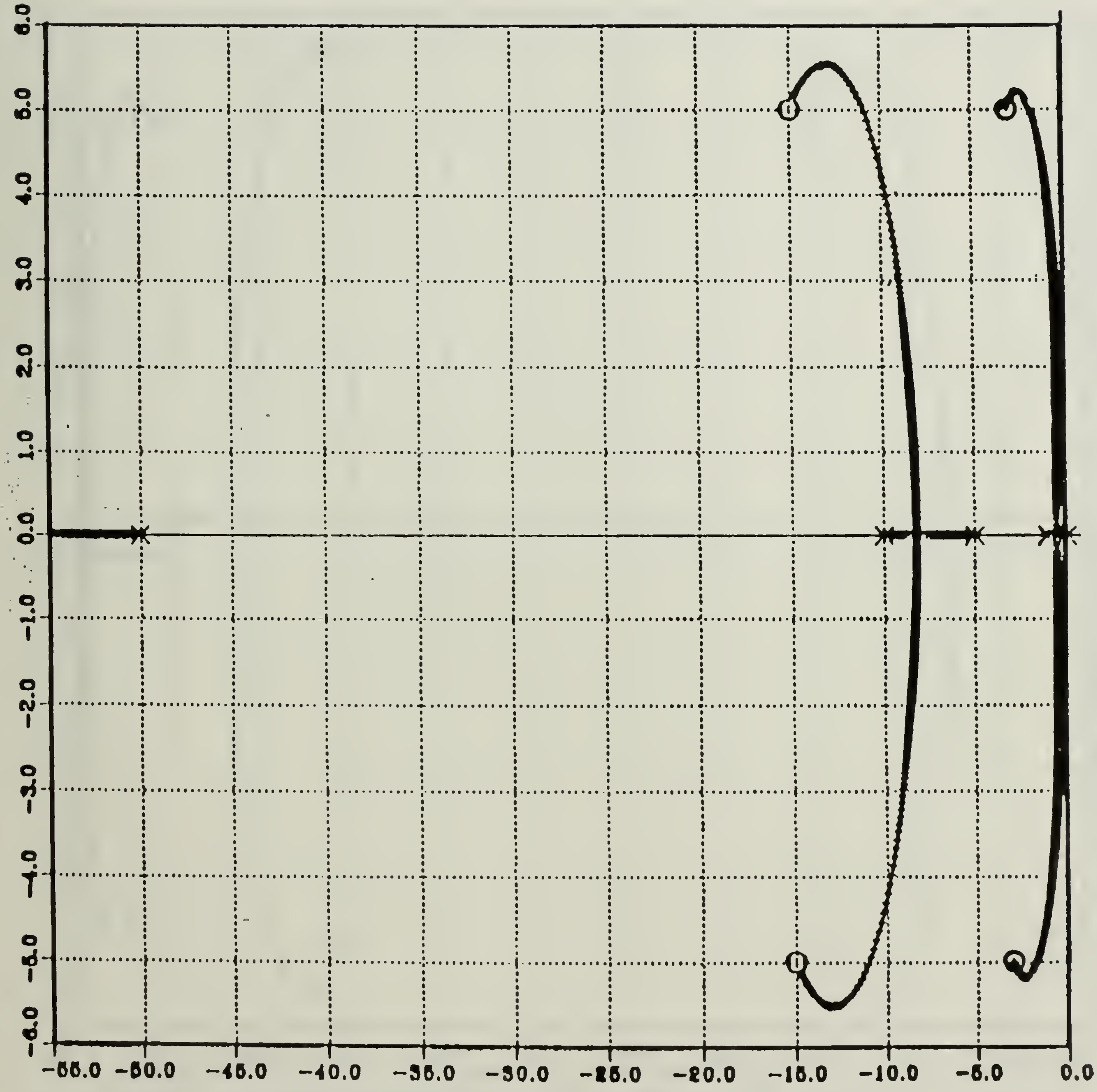

Figure 54. Original Compensated System Root Loci. Desired Roots; -3.000 ∓ j5.000 , -15.000 ∓ j5.000

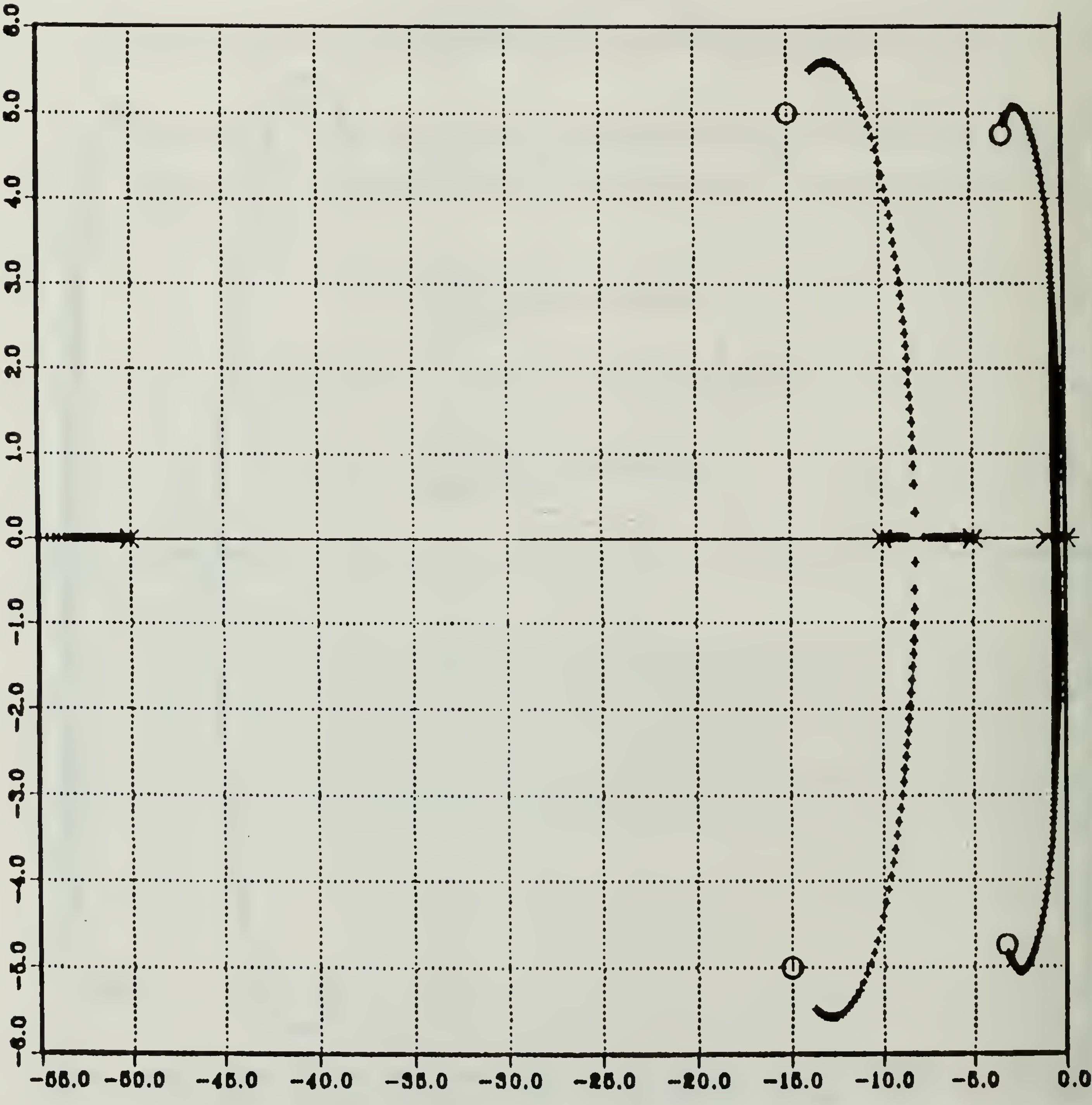

Figure 55. Offsetting The Zeros For The First Time. Desired Roots; -3.000 ∓ j5.000 , -15.000 ∓ j5.000

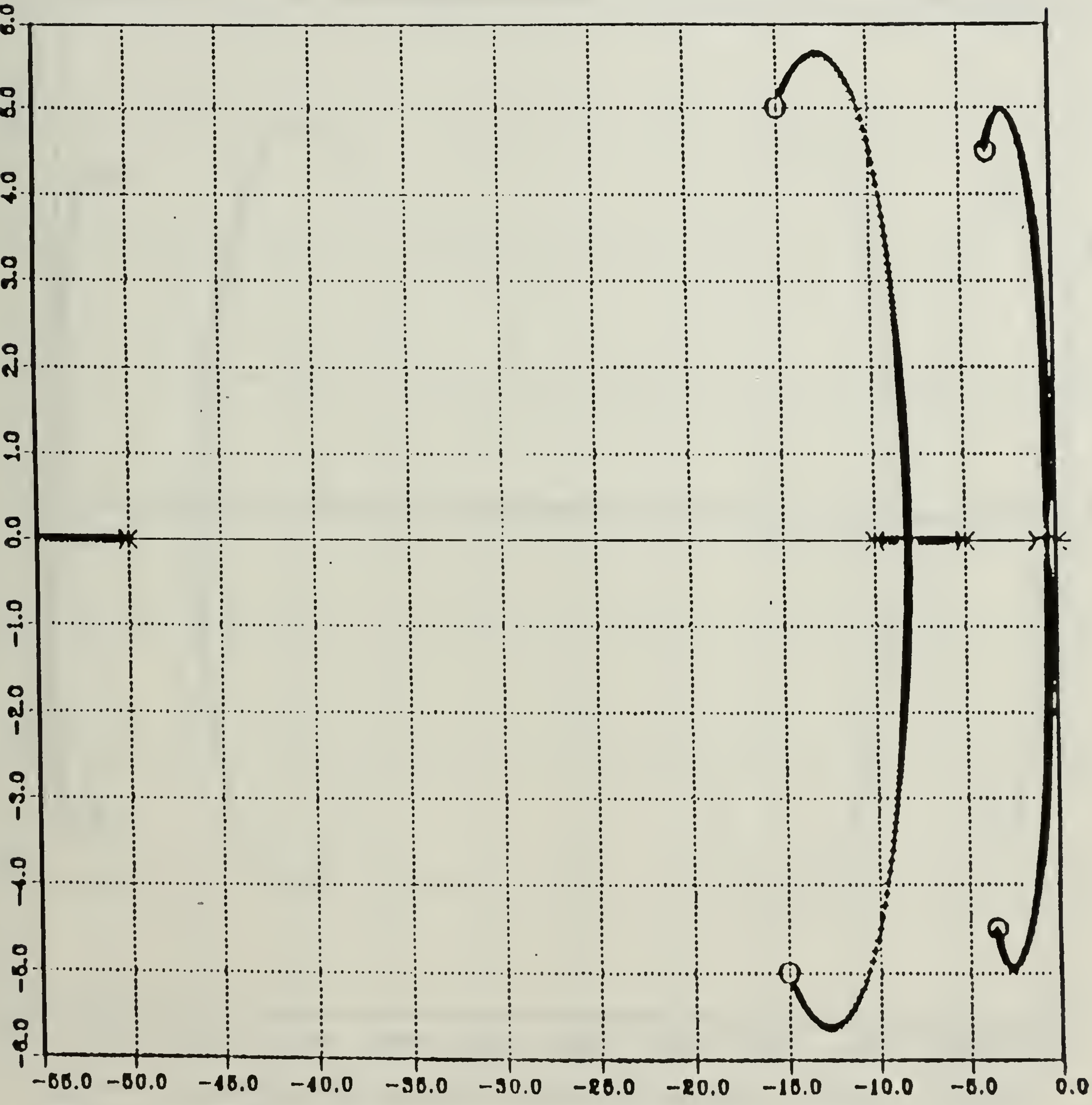

Figure 56. Offsetting The Zeros For The Second Time. Desired Roots; -3.000 ∓ j5.000 , -15.000 ∓ j5.000

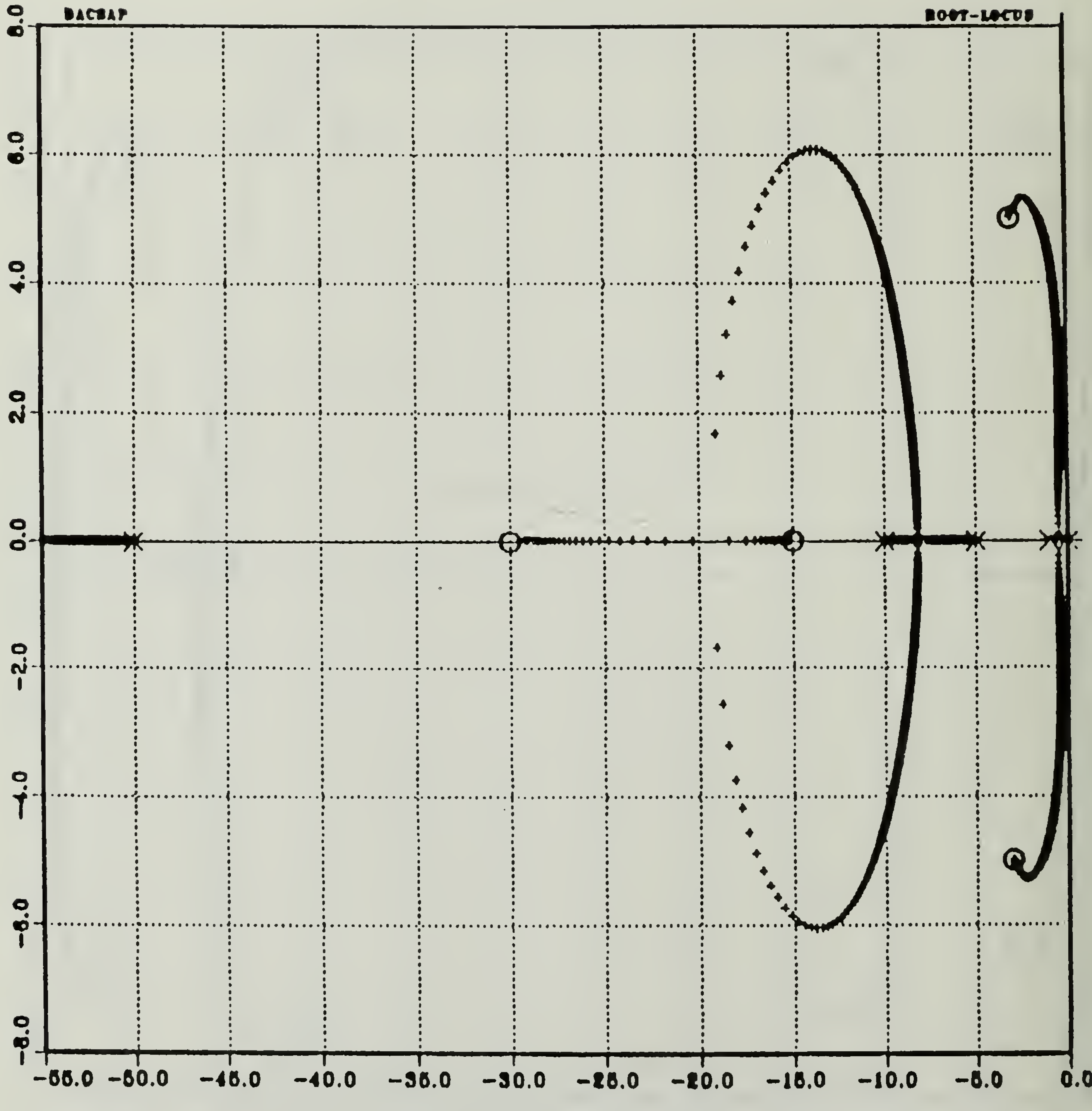

Figure 57. Original Compensated System Root Loci. Desired Roots; -3.000 ∓ j5.000 , -15.000 , -30.000

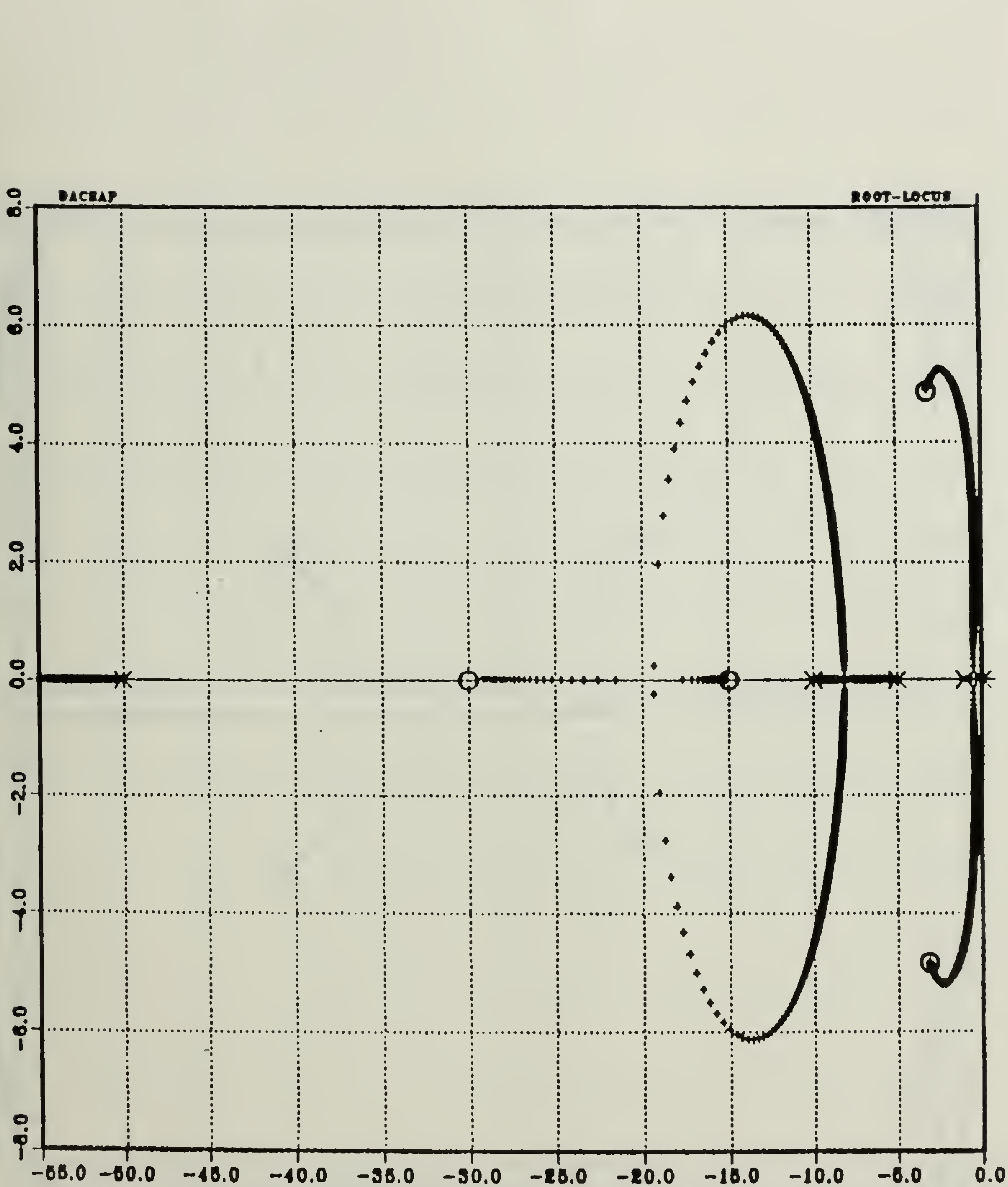

Figure 58. Offsetting The Zeros. Desired Roots; -3.000 ∓ j5.000 , -15.000 , -30.000

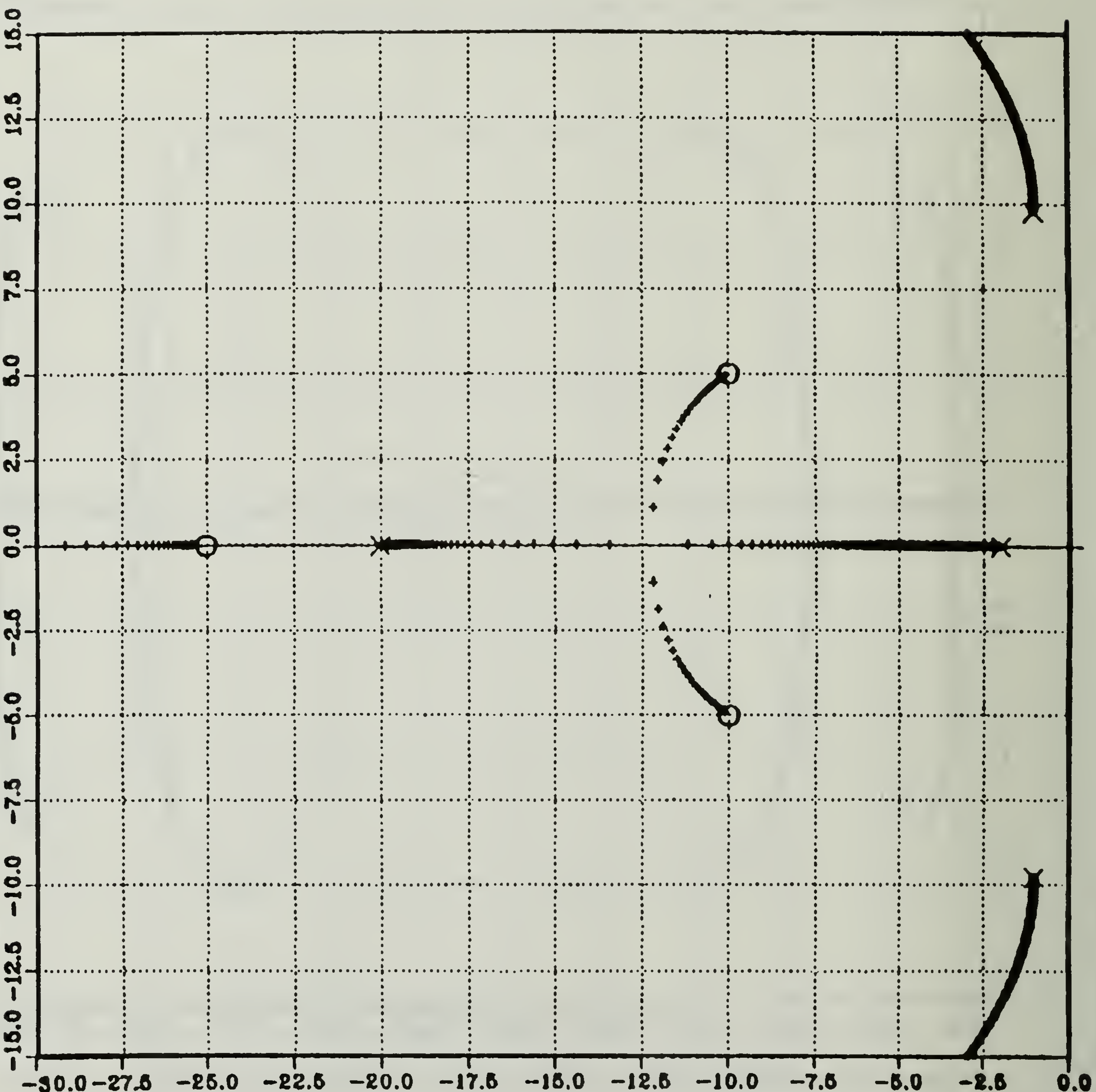

Figure 59. Original Compensated System Root Loci. Desired Roots; -10.000 ∓ j5.000 , -25.000

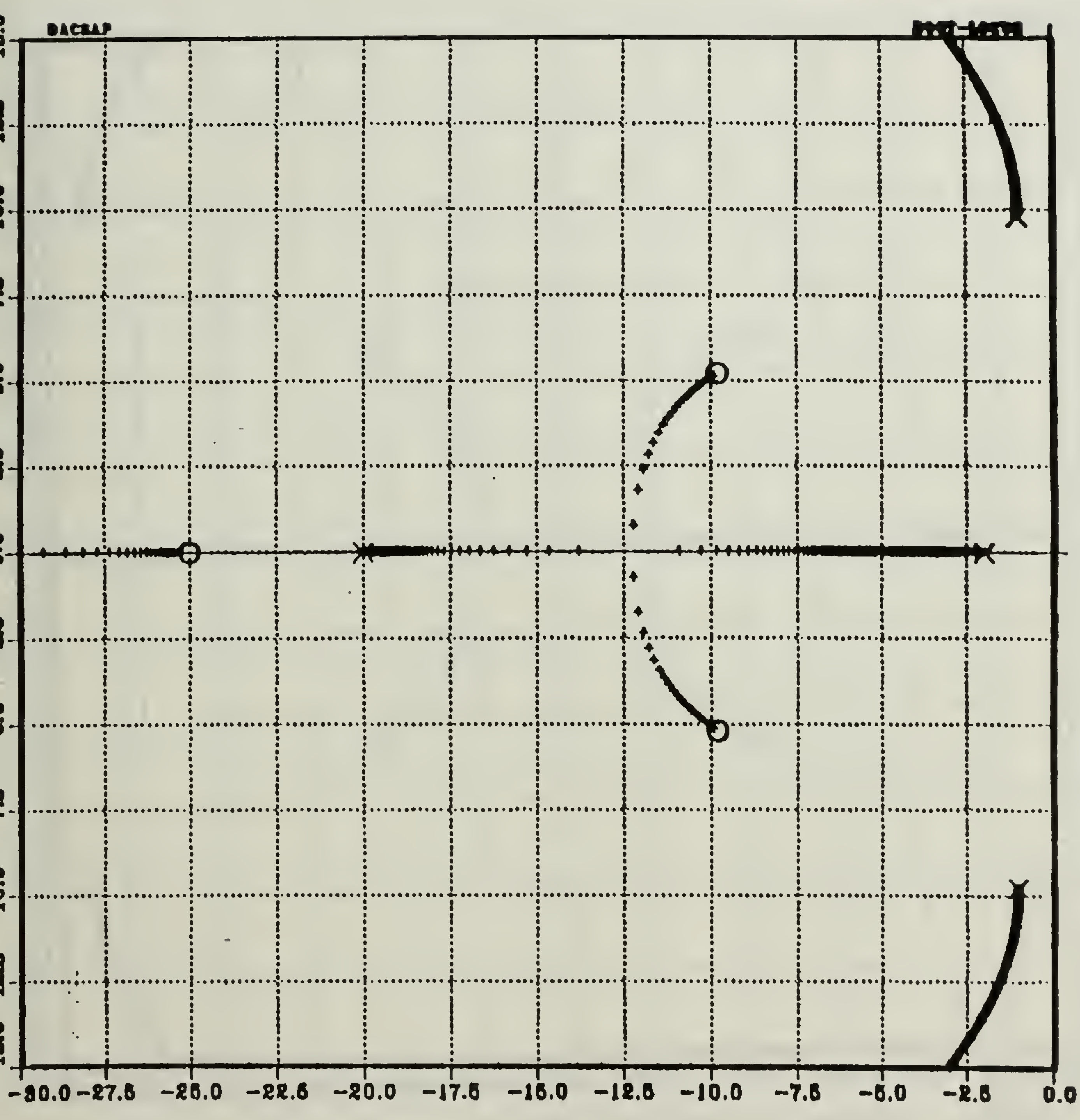

Figure 60. Offsetting The Zeros For The First Time. Desired Roots; -10.000 ∓ j5.000 , -25.000

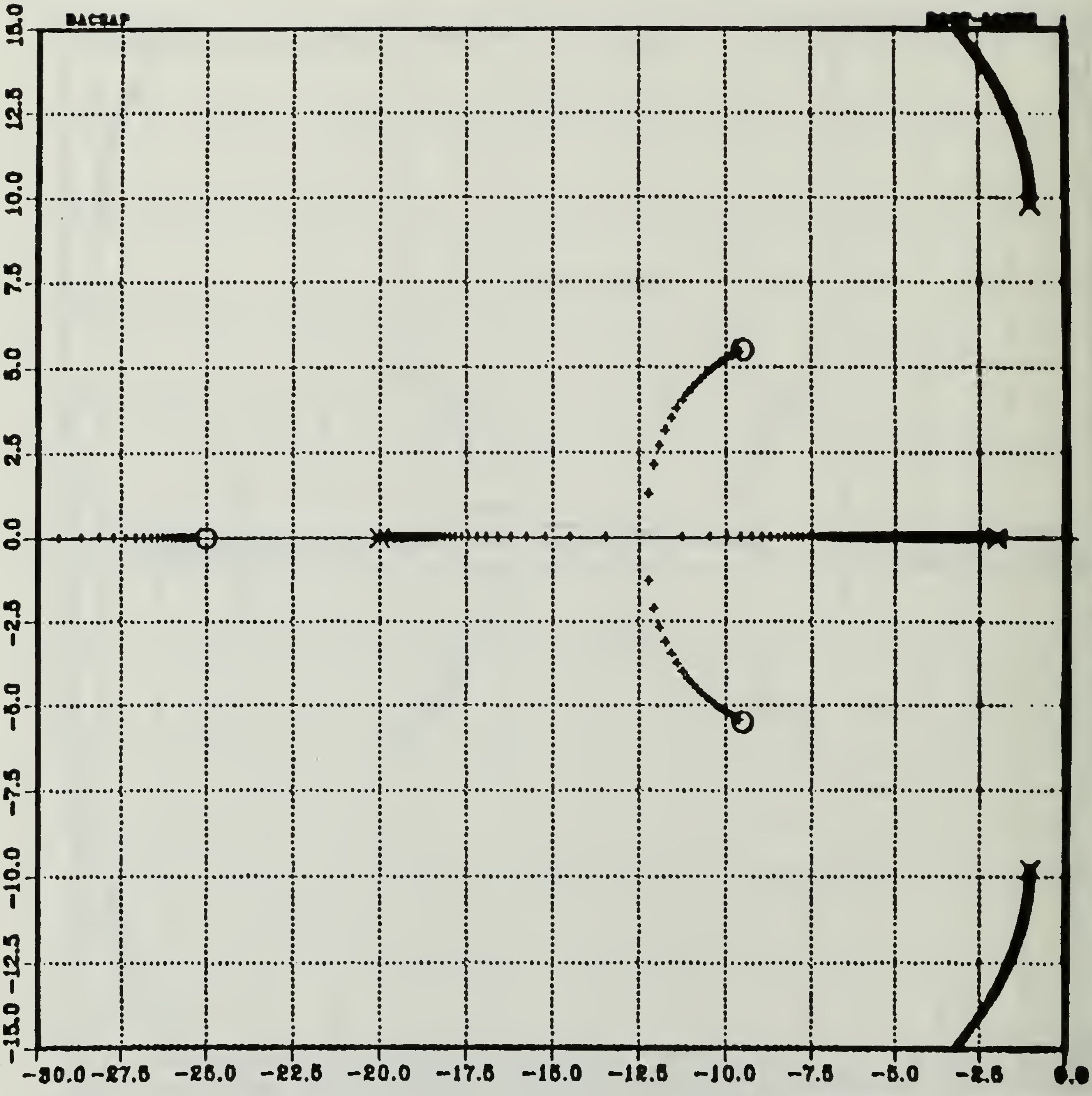

Figure 61. Offsetting The Zeros For The Second Time. Desired Roots; -10.000 ∓ j5.000 , -25.000

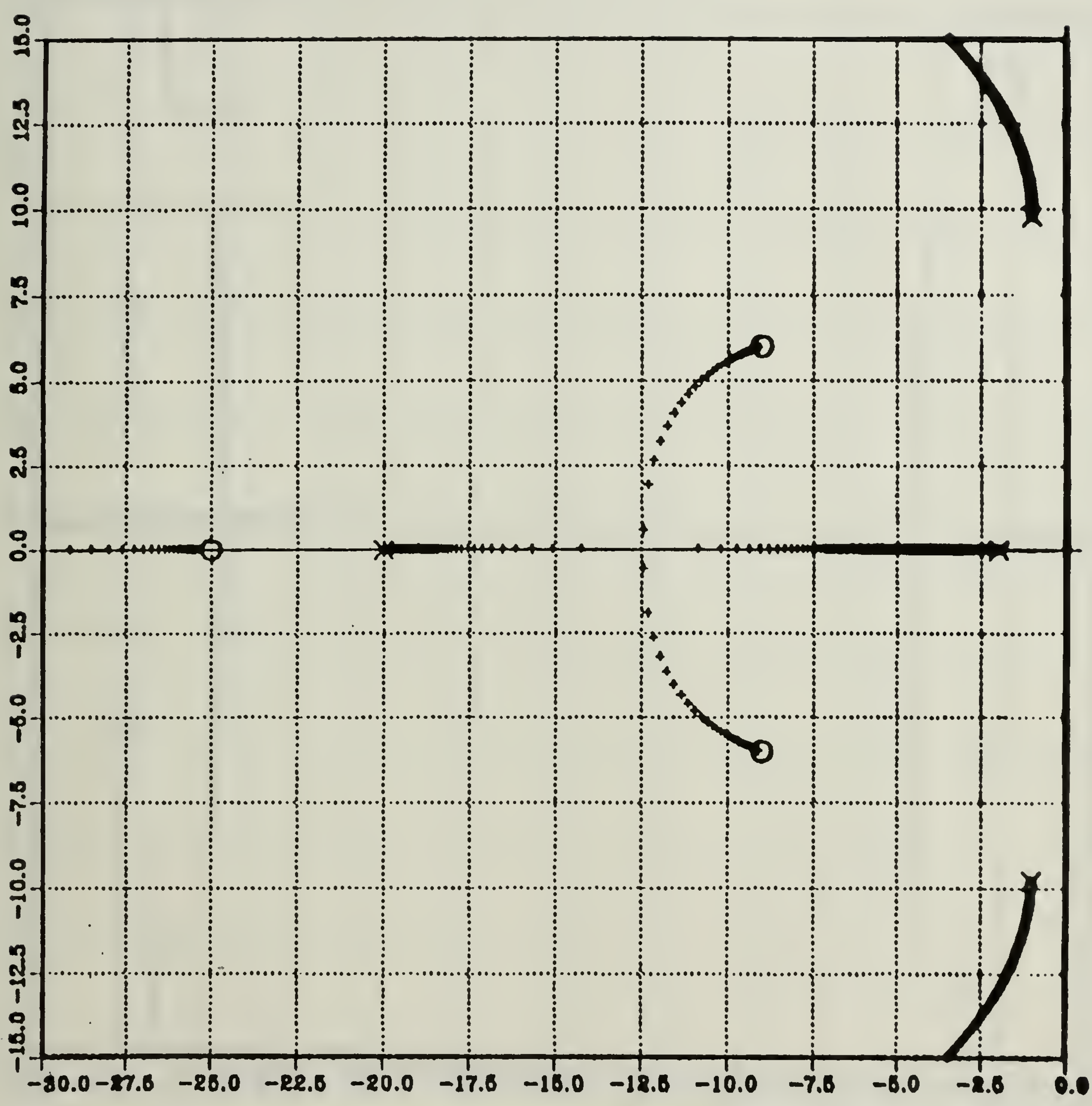

Figure 62. Offsetting The Zeros For The Third Time. Desired Roots; -10.000 ∓ j5.000 , -25.000

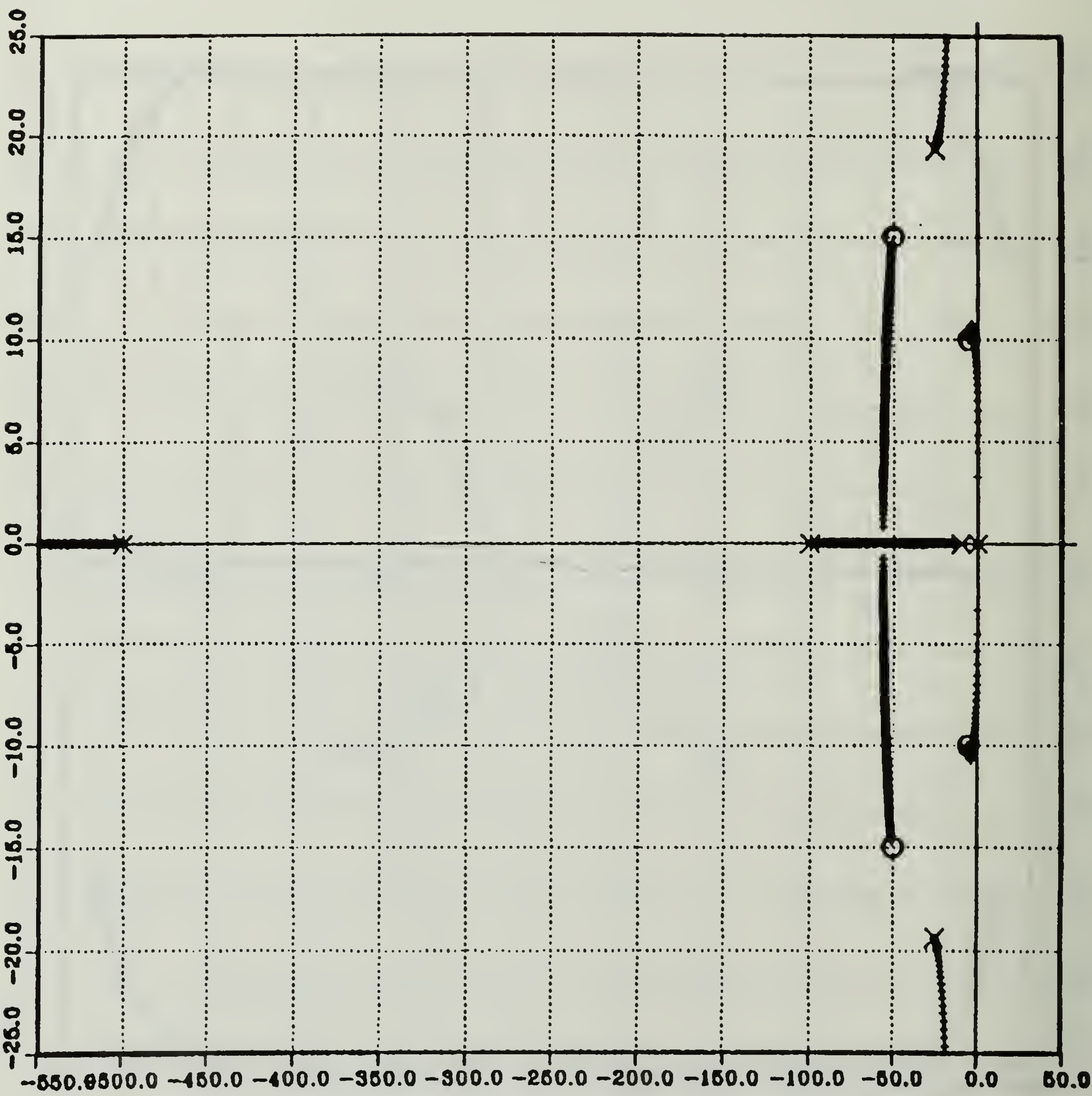

Figure 63. Original Compensated System Root Loci. Desired Roots; -5.000 ∓ j10.000 , -50.000 ∓ j15.000 , -300 ∓ j200

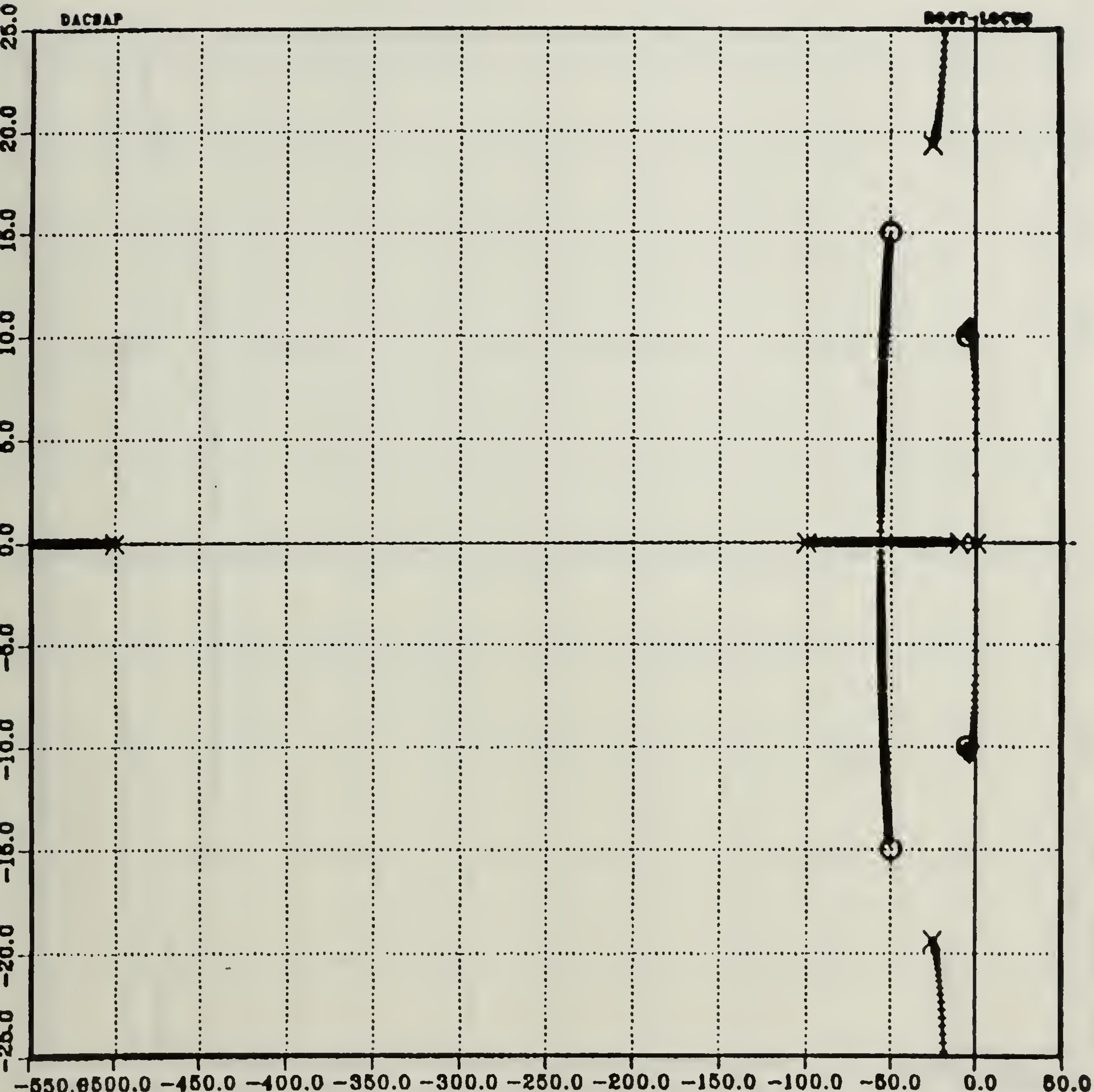

Figure 64. Offsetting The Zeros For The First Time. Desired Roots; -5.000 ∓ j10.000 , -50.000 ∓ j15.000 , -300 ∓ j200

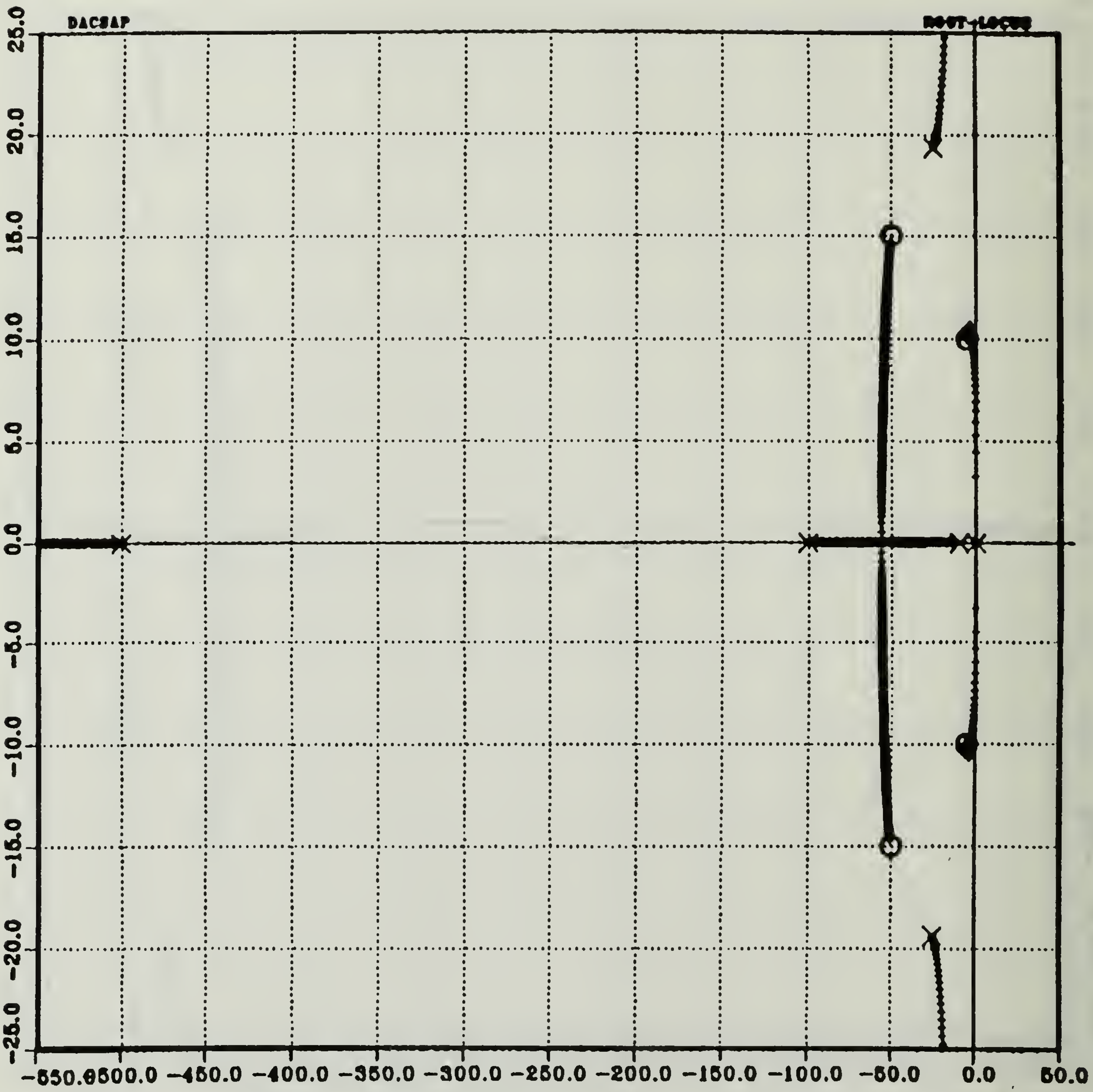

Figure 65. Offsetting The Zeros For The Second Time. Desired Roots; -5.000 ∓ j10.000 , -50.000 ∓ j15.000 , -300 ∓ j200

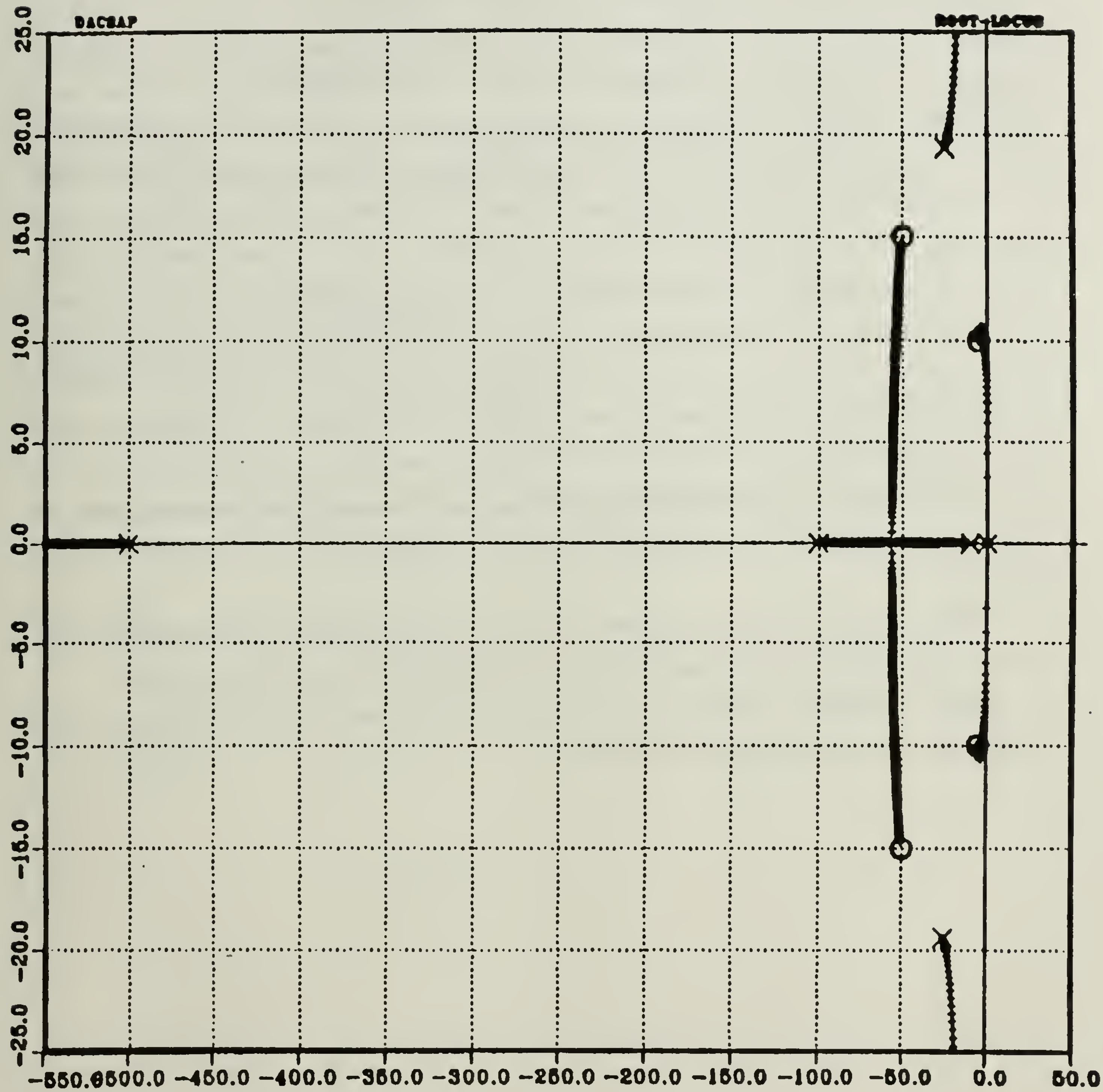

Figure 66. Offsetting The Zeros For The Third Time. Desired Roots; -5.000 ∓ j10.000 , -50.000 ∓ j15.000 , -300 ∓ j200

VI. CONCLUSIONS AND RECOMMENDATIONS

A transfer function approach to root placement using state feedback is a very useful and effective design tool for modern automatic control systems. When all N states are available to be measured and fedback, the full state feedback design procedure gives good results to satisfy required system time performance and bandwidth specifications. It is assumed that the designer knows how to choose the N-1 desired roots which define H(s) such that the system specifications are met. After choosing H(s), it should be used to plot the root loci for G(s)H(s) to inspect possible locations of the closed loop roots as a function of gain, K. If the root loci do not traverse suitable locations in the s-plane, or if they do but extremely high gain is needed, then the dominant zeros should be relocated. The root loci are extended from the zeros in the natural direction. If they do not pass through the desired points, new offset points are chosen by moving the zeros until the root loci do pass though the desired points or at an acceptable distance from the desired points. The guideline that estimates the decrease of the loop gain may not work for some plants. Then offsetting the zeros is repeated until an acceptable result is obtained.

If fewer than N states are available to be measured and fedback, these methods may be used with partial state feedback, but a successful result cannot be guaranteed.

The SVS computer program for root placement by using matrix methods is not as flexible as the transfer function methods.

APPENDIX THE ERROR COEFFICIENTS FOR THE TRANSFER FUNCTIONS IN BODE FORM

$$G_{eq}(s) = \frac{K_v \prod_{j=1}^{m} (\tau_j + 1)}{s^N \prod_{i=1}^{n} (\tau_i + 1) + K_v \prod_{j=1}^{m} (\tau_j s + 1) \sum_{i=1}^{n+N} k_x s^x}$$

$$G_{net}(s) = \frac{k_0 K_v \prod_{j=1}^{m} (\tau_j s + 1)}{s_N \prod_{i=1}^{n} (\tau_i s + 1) + K_v \prod_{j=1}^{m} (\tau_j s + 1) \sum_{i=1}^{n+N} k_x s^x}$$

For Different Type Systems The Error Coefficients are:

$K_p = k_0 K_v$ $\quad N = 0$

$$K_v = \frac{k_0 K_v}{1 + K_v k_1} \qquad N = 1$$

$K_a = 0$ $\quad N = 2$

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